California State University, Long Beach
Department of Mathematics and Statistics

Syllabus for the Real Analysis Comprehensive Examination

1. Topics

   (a) Algebras and σ-algebras of sets; countable sets.

   (b) Real and extended real numbers; open, closed, and compact sets; structure of open sets; continuity of real functions.

   (c) Lebesgue outer measure on $\mathbb{R}$; Lebesgue measurable sets; Lebesgue measure; F_σ, G_σ and Borel sets and their relation to Lebesgue measurable sets; the Cantor set.

   (d) Lebesgue measurable functions; approximation by simple functions, step functions and continuous functions, Lusin's Theorem.

   (e) The Lebesgue integral: definition, basic properties, relation to Riemann integral, convergence theorems.

   (f) Convergence: pointwise, a.e., uniform, almost uniform, in mean, in measure; implications between modes of convergence; Egoroff's Theorem.

   (g) Differentiation: Vitali's Lemma, monotone functions and functions of bounded variation; indefinite integrals; absolute continuity.

2. References

   (a) Royden, *Real Analysis*, Macmillan, 1968: Ch. 1–6.

   (b) Rolland, *Real Analysis*, McGraw-Hill, 1986: Ch. 1–2, sec. 3.5, 6.1.

   (c) Redin, *Principles of Math Analysis*, 3rd edition, McGraw-Hill, 1976: Ch. 10.

   (d) Rudin, *Real and Complex Analysis*, McGraw-Hill, 1966: Ch. 1–3.

   (e) Wheeden and Zygmund, *Measure and INtegral*, Dekker, 1977: Ch. 1–7.

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