Master's Comprehensive Exam – Experimental Design NAME: ___________________________

Instructions: Solve *five* out of the *seven* problems given below.

Circle *five* problems chosen: 1 2 3 4 5 6 7

February 16, 2013

1. Suppose we wish to investigate differences in test scores for three selected schools and two named teachers. Each teacher teaches two different classes.

		Teacher	
		1	**2**
School	**1**	25	14
		29	11
	2	11	22
		6	18
	3	17	5
		20	2

   a) Assume that we are interested in the specific schools and the named teachers. State the model, conduct the ANOVA and draw an appropriate conclusion at the 5% significance level.
   b) Now, interest could be in comparing schools 1 and 2. Find a 95% confidence intervals (CI) for the mean test score for school 1 and a 95% CI for the mean difference in test scores between schools 1 and 2.
   c) Now you assume that the named teachers are randomly selected. Repeat (b).

2. A study is made on the yield of a product manufactured on two different machines at three different power settings. Due to the technical difficulties, the order of runs within a lab cannot be completely random. Because of the long warm-up period and the related costs, once the machine is on, the three measurements for all three power settings must be taken sequentially. The measurements were taken over three randomly selected labs (block).

	Lab 1		**Lab 2**		**Lab 3**	
Machine	1	2	1	2	1	2
Power Setting						
P1	34	33	28	26	49	42
P2	14	12	19	17	25	25
P3	26	14	20	15	28	24

   a) Specify the design and write the statistical model with all specifications. Estimate all the parameters in the model.
   b) Give the ANOVA table and draw conclusions.
   c) Construct the confidence interval comparing power settings using Tukey's method.

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3. Answer the following questions.
   a) In the two-factor (A and B) mixed model analysis of variance, write down the restricted model with assumptions. Specify what is restricted and interpret the meaning. Show that $Cov\left[(\tau\beta)_{ij}, (\tau\beta)_{i'j}\right] = -\frac{1}{a}\sigma^2_{\tau\beta}$ for $i \neq i'$.
   b) Consider the three-factor (A, B and C) factorial model for a single replicate. Assume that all factors are random. Write down the model. State expected mean squares for each model term. Provide the test statistic to test the main effect of A.
   c) Consider the model in (b). Assume that the three-factor interaction and AB interaction do not exist. Write the model, and develop a skeleton ANOVA table (Source and DF), including the expected mean squares. Now provide the test statistic to test the main effect of A. Also provide the test statistics to test the main effect C.

4. An experiment was performed to improve the yield of an agricultural process. Three factors (A, B, C) were selected and each factor is run at two levels, and the design is replicated twice. The result is shown below.

	A1				**A2**			
	B1		**B2**		**B1**		**B2**	
C1	52	60	68	60	69	65	61	65
C2	101	105	108	106	77	87	70	86

   a) Identify the design and perform the hypothesis tests for all the factors including interactions.

   Suppose that the experiment is done over two different locations and only four treatment combinations can be done at a location at a time. Since there could be a location effect in the yield, the experimenter decided to use location as a blocking factor. For the fist replicate the factor combination (1,1,1), (1,1,2),(2,2,1), and (2,2,2) have been done at the first location and other combinations at the second location. For the second replicate the factor combination (1,1,1), (1,2,2),(2,2,1), and (2,1,2) have been done at the first location and other combinations at the second location. Here (1,1,1) means (A1, B1, C1) treatment combination.

   b) Identify the design and find the confounded factor for each replicate. Redo the hypothesis tests. Compare the result with (a).

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5. A researcher conducted an experiment to compare the effects of three different insecticides on a variety of string beans. To obtain a sufficient amount of data, it was necessary to use four randomly selected plots of land. Since the plots had somewhat different soil fertility, drainage characteristics, and sheltering from winds, the researcher decided to conduct a randomized complete block design with the plots serving as the blocks. Each plot was subdivided into three rows. A suitable distance was maintained between rows within a plot so that the insecticides could be confined to a particular row. The insecticides were randomly assigned to the rows within a plot so that each insecticide appeared in one row within all four plots. The response of interest was the number of seedlings that emerged per row. The data are given below.

Insecticide		**1**	**2**	**3**
Plot	**P1**	56	83	80
	P2	48	78	72
	P3	66	94	83
	P4	62	93	85

a) Write the statistical model with all the specifications. Estimate all the parameters in the model including variances.
b) Briefly plot the interaction plot and validate the model in (a).
c) Give ANOVA table and perform the hypothesis test for the Insecticide effect and draw a conclusion.
d) Use the Fisher's method for all pairwise comparisons between insecticides.

6. Answer the following design questions.
a) Construct the 2_{III}^{7-4} design. Determine aliases for main effects and two-way interaction effects.
b) Fold over the 2_{III}^{7-4} design to produce 2_{IV}^{8-4}design.
c) Verify that the resulting design in (b) is actually 2_{IV}^{8-4}design by providing a design with generators. Is 2_{IV}^{8-4} a minimal design?

7. Consider 2^3 factorial design with three replicates. Data shows life span in years of Gorillas.

$$(1) = 22, 31, 25; a = 32, 43, 29; b = 35, 34, 50; ab = 55, 47, 46; c = 44, 45, 38;$$
$$ac = 40, 37, 36; bc = 60, 50, 54; abc = 39, 41, 47$$

Hint) SST = 2095.33.
a) Conduct ANOVA and construct the table. Based on the analysis of main effects and interaction plots, what levels of A, B, and C would you recommend using?
b) Based on the normal probability plot of effects below, construct the reduced model ANOVA table. Conduct the Lack of Fit (LOF) test. Interpret the results from the LOF test.
c) Suppose that we have confounded ABC in replicate I, AB in replicate II, and BC in replicate III. Construct the ANOVA table.

STOP HERE!!