California State University, Long Beach
Department of Mathematics and Statistics

Syllabus for the Complex Analysis Comprehensive Examination

1. Topics

   (a) *Complex numbers*: the algebra of complex numbers, geometric representation of complex numbers, the complex plane as a metric space, the extended complex plane, spherical representation.

   (b) *Complex functions*: limits, continuity, analyticity, rational functions, expenential functions, trig functions, branches of logarithm.

   (c) *Analytic functions*: harmonic functions, harmonic conjugates, Cauchy-Riemann equations, power series representation.

   (d) *Conformal mappings*: mapping properties of elementary functions, linear (fractional) transformations, cross ratio, symmetry and orientation principles, Schwarz's lemma.

   (e) *Complex integration*: line integral, index or winding number, Cauchy's theorem, Cauchy's interal formula, Cauchy's estimate, Morera's theorem, Goursat's theorem.

   (f) *Zeros of analytic functions*: entire functions, Liouville's theorem, fundamental theorem of algebra, open mapping theorem, maximum modulus theorem.

   (g) *Singularities*: removable and essential singularities, poles, Laurent series, residues, residue theorem, calculus of residues, meromorphic functions, argument principle, Rouche's theorem.

2. References

   (a) Ahlfors, *Complex Analysis*, 2nd ed., McGraw-Hill, 1966, Ch. 1–4.

   (b) Conway, *Functions of One Complex Variable*, 2nd ed., Springer-Verlag, 1978, Ch. 1–6.

   (c) Hille, *Analytic Function Theory*, vol. I, Ginn, 1959.

   (d) Greenleaf, *Introduction to Complex Variables*, Saunders, 1972.

   (e) Heins, *Complex Function Theory*, Academic Press, 1968.

   (f) Curtiss, *Introduction to Functions of a Complex Variable*, 1978.

Revised 9-99.