California State University, Long Beach
Department of Mathematics and Statistics

**Syllabus for the Algebra Comprehensive Examination**

1. Topics

   (a) Groups

   - Elementary properties of groups
   - Subgroups
   - Cyclic groups
   - Cosets, indices of subgroups, Lagrange's Theorem, counting
   - Homomorphisms of groups; kernel, image
   - Normality, quotient groups, isomorphism theorems
   - Symmetric and alternating groups
   - Group action on a set: orbits, stabilizers
   - Permutation representations of groups
   - Conjugacy classes, the class equation
   - Automorphisms of groups
   - The Sylow Theorems
   - Free abelian groups, finitely generated abelian groups

   (b) Rings

   - Rings, integral domains, division rings, fields
   - Characteristic of a ring
   - Subrings, ideals, quotient rings
   - Homomorphisms of rings, isomorphism theorems
   - Characterization of prime and maximal ideals
   - Direct product of rings
   - Principal ideal domains, unique factorization domains, Euclidean domains

   (c) Linear Algebra

   - The content of the undergraduate linear algebra course MATH 247, including matrix algebra, vector spaces, linear transformations, eigenvalues and eigenvectors, similarity and diagonalization of matrices

2. References


   (a) Bhattacharya, Jain, and Nagpaul, *Basic Abstract Algebra*, Cambridge, 1994

   (b) Dummit and Foote, *Abstract Algebra*, Prentice Hall, 2004

   (c) Fraleigh, *A First Course in Abstract Algebra*, Addison-Wesley, 2003

   (d) Isaacs, *Algebra: A Graduate Course*, Brooks/Cole, 1994

   (e) Snaith, *Groups, Rings and Galois Theory*, World Scientific, 1998


Last revised 5-2007 by Dr. Robert Valentini and Dr. Will Murray.