## *Demo 03-19*   **Stability**

This demonstration illustrates the concepts of stable, neutral, and unstable equilibrium, using a cube, a sphere, and a cone, respectively, as illustrated in *Figure 1*.[†] The cube is in stable equilibrium, and returns to its original position if it is lifted slightly along one edge. The sphere at rest is in neutral equilibrium, and will remain at rest if given a slight displacement. The cone is in unstable equilibrium if carefully balanced on its tip, and will fall over if displaced very slightly from that position.
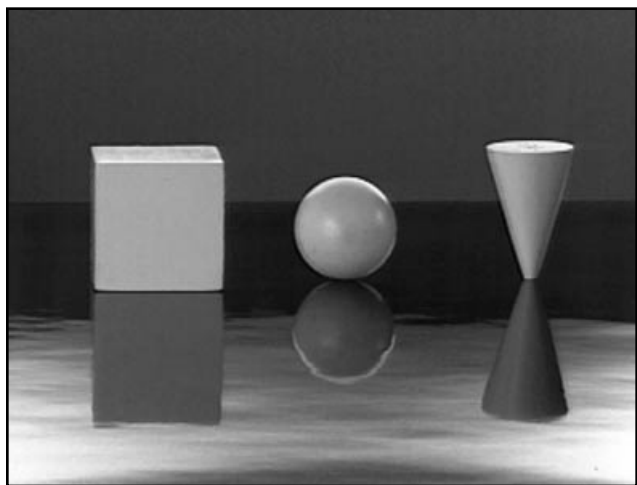
Figure 1

[†] Freier and Anderson, *A Demonstration Handbook for Physics,* Demonstration Mq-2, Stability of Cone and Sphere.

We'll use these objects to demonstrate three different types of equilibrium. This cube is in stable equilibrium. If it is displaced by a small amount, it returns to its original orientation.

This sphere is in neutral equilibrium. It is stable in its original orientation, and in any other orientation.

This cone is in unstable equilibrium.

If it is displaced by even a small amount, it falls over instead of returning to its original orientation.

This animation shows the weight that acts on the center of mass of each of the objects as it is tipped.

Equipment

1. A cube.
2. A sphere.
3. A cone.

Irregular Object Center of Mass

When a plane irregular object is suspended from any point on the object, it will hang such that the center of mass is directly below the point from which it is suspended.[†] An irregular object is suspended from two such points, locating the center of mass, as shown in *Figure 1*. When the object is then suspended from its center of mass, it is stable in any orientation, as shown in the video.

Figure 1

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† Sutton, *Demonstration Experiments in Physics,* Demonstration M-31, Center of Gravity, and M-32, Equilibrium.
Freier and Anderson, *A Demonstration Handbook for Physics,* Demonstration Mp-7, Irregular Shape.

Is there a simple way to locate the center of mass of an irregular flat object?

We'll hang the sheet from a support point near its edge. Since it is in stable equilibrium, we know that its center of mass is directly below the point of support. A plumb bob is hung from the same support point, and we snap the chalk-filled string to put a line on the sheet.

Then we repeat the same action from another point of support.

Since the center of mass must be located on both these lines, it must be at their intersection.

If we suspend the sheet from the intersection point, it is stable in any orientation.

This animation shows the forces acting on the sheet, and the net restoring force that results when the center of mass is not below the point of support.

Equipment

1. Tall ring stand.
2. Support rod.
3. Irregular shaped body.
4. Plumb bob on string.
5. Soft chalk supply.

Demo 03-21   **Center of Mass Disc**

For a complex object in a gravitational field, the weight of the object—the gravitational force on the object—acts as though it were concentrated at the center of mass.[†] This means that if the object rotates as it flies through the air, its center of mass will move in a parabolic path, the same as that of a small but massive projectile. A disc is thrown through the air, first with its center of mass at the center of the disc, then with its center of mass displaced from the center of the disc, and the paths of the center of mass are traced out in each case. The latter case is illustrated in *Figure 1*.

Figure 1

This symmetrical foam disc has a heavy weight at its center. The center of mass is therefore at the center of the disc. When the disc is thrown, that point moves in a smooth parabola.

When the weight is shifted to the edge of the disc, the center no longer moves smoothly.

This point marks a new center of mass of the disc. When the disc is thrown, that point now moves in a smooth parabola.

Equipment

1. Cardboard or Styrofoam disc.
2. Disc weight to move from center on one side of disc to the near edge position on the opposite side.
3. Mark the new center of mass to aid visibility.

Demo 03-22   **Chair on Pedestal**

An upright chair is balanced on a vertical rod supporting the chair at a point under the seat at the center of the legs, as shown in *Figure 1*.[†] This seemingly impossible equilibrium position is achieved by placing heavy weights in holes strategically drilled in the ends of the chair legs, thus lowering the center of mass below the point at which the chair is balanced.

Figure 1

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† Freier and Anderson, *A Demonstration Handbook for Physics,* Demonstration Mp-12, Center of Gravity of a Stool.

This chair can be mounted on a sharp point at the top of this bar.

The chair is now stable in all orientations and can be swung vigorously without tipping from support. Why is the chair so stable?

Weights have been added inside the legs, which lowers the center of mass to below the point of support.

Equipment

1. Vertical support rod with point and heavy base.
2. Wooden chair—optimize by presetting dimple plate on the underside of the chair's seat and implanting weights within each of the chair's legs.

A toy clown holding a rod with weights on its ends, like the traditional tightrope walker, rolls upright on a pulley along a rope, as shown in *Figure 1.*[†] It remains stable because the weights lower the center of mass to a point below the point at which the pulley contacts the rope. When the rod and weights are removed from the grasp of the clown, its center of mass is above the contact point and it becomes unstable.

Figure 1

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† Freier and Anderson, *A Demonstration Handbook for Physics,* Demonstration Mp-6, Unicycle.

This toy clown is stable when placed on a tight string.

Why doesn't the clown fall off the string?

These weights shift the clown's center of mass to a point below the wheel. If we remove the weights, we shift the center of mass to a point above the wheel. Here's what happens without the weights.

With its center of mass above the wheel, the clown is no longer stable.

Equipment

1. Toy clown on unicycle with balance bar and counterweights.
2. Long length of string.
3. Place to tie far end of string or an assistant to hold it.

Demo 03-24   **Double Cone on Incline**

A double cone rolls on a pair of rails, as shown in *Figure 1*, in such a way as to appear to roll uphill.[†] In reality the center of mass becomes lower as the double cone rolls along the rising rails, because of the geometry of the system. A cylinder rolls downhill, as expected.

Figure 1

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† Sutton, *Demonstration Experiments in Physics,* Demonstration M-37.
  Freier and Anderson, *A Demonstration Handbook for Physics,* Demonstration Mr-1, Rolling Uphill.

When we place a cylinder on this pair of inclined rails, it rolls down the incline.

If we place this double cone on the incline, it rolls up the incline.

Why does this object appear to defy gravity? The rails spread apart in the uphill direction.

This and the shape of the double cone allows the center of mass of the cone to go lower as it moves "up" the rails.

Here's a shot from the side.

Equipment

1. Inclined rails whose distance of separation decreases to a near point at opposite lower end.
2. Cylinder.
3. Double cone.

A wooden disc is loaded with a heavy weight near its perimeter, causing the center of mass of the disc to be located well off the center of the disc. By carefully positioning the disc on an incline, we can then make it roll both down and *up* the incline,[†] as shown in *Figure 1*.

Figure 1

[†] Sutton, *Demonstration Experiments in Physics,* Demonstration M-35.
Freier and Anderson, *A Demonstration Handbook for Physics,* Demonstration Mp-3, Disc Rolling Uphill.

When we place this disc on an incline, it rolls downhill.

If we place the same disc on the incline again, it rolls uphill. What could account for this unusual behavior?

If we turn the disc around, the reason becomes clear.

A heavy weight has been added to the disc near its edge, shifting the center of mass. The center of mass of the disc can drop as the disc rolls uphill.

Equipment

1. Low angle incline plane.
2. Heavy wooden disc that bears a massive slug implant near its edge.

Demo 03-26   Toppling Cylinders

This demonstration illustrates the effect of the center of mass and torques using two "toppling cylinders," constructed from aluminum tube, in a surprising way.[†] *Figure 1* illustrates the two toppling cylinders.

The first cylinder, at the right in *Figure 1*, is a standard "leaning tower" demonstration in which the addition of a cap to a tilted cylinder moves the center of mass outside of the volume directly above its base, causing the cylinder to topple.[‡] The second cylinder stands vertically with its cap on, but topples when the cap is removed. This is due to two balls of the appropriate mass and radius that are properly positioned in the tube with the cap on. For the vertical cylinder to topple when its cap is removed, the relationship between the radius r_1 of the lower ball, the mass M_2 and the radius r_2 of the upper ball, and the mass M and inside radius R of the cylinder is:

$$M < M_2 \left\{ 2 - \left[\frac{r_1 + r_2}{R} \right] \right\}$$

In the case on the video $r_1 = r_2 = 1.9\,\text{cm}$, $R = 2.5\,\text{cm}$, $M_2 = 230.5\,\text{g}$, $M = 68.5\,\text{g}$, and the mass of the cap is $85.5\,\text{g}$.
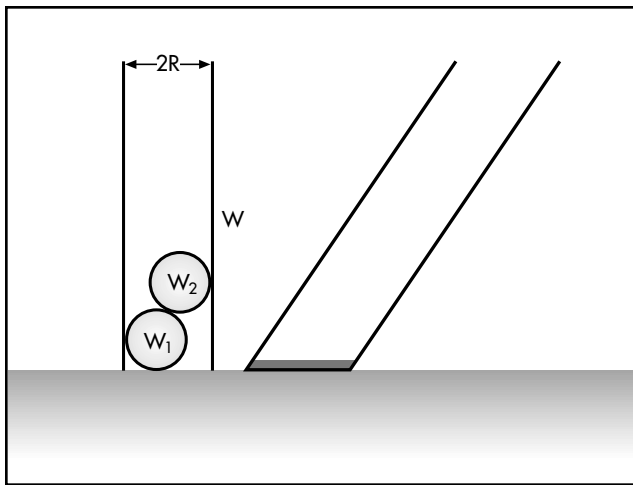

Figure 1

[†] M.E. Gardner, Falling Cylinders, *Am. J. Phys.* 34, 822 (1966).

[‡] Sutton, *Demonstration Experiments in Physics,* Demonstration M-34.
Freier and Anderson, *A Demonstration Handbook for Physics,* Demonstration Mp-9, Leaning Tower of Pisa.

Here are two aluminum cylinders.

The tilted cylinder topples when its cap is removed.

Inside the vertical cylinder are two balls with the same radius, a Ping-Pong ball on the bottom and a steel ball on the top.

The torque on the aluminum tube caused by heavy ball on top is sufficient to tip the tube without the weight of the cap.

Here is what happens if the lighter ball is put on top.

Equipment

1. Aluminum cylinder fitted with a cap and a hollow lower half.
2. Aluminum cylinder fitted with a cap and its two ends cut at an angle whereby it tips over with the addition of its cap.
3. A Ping-Pong ball.
4. A steel sphere the same size as the Ping-Pong ball.

Demo 03-27   **Air Table Center of Mass**

A long flat piece of wood, weighted at one end, is slid across an air table while it is rotating. The center of mass moves in a straight line, as shown in *Figure 1*, taken from the video.
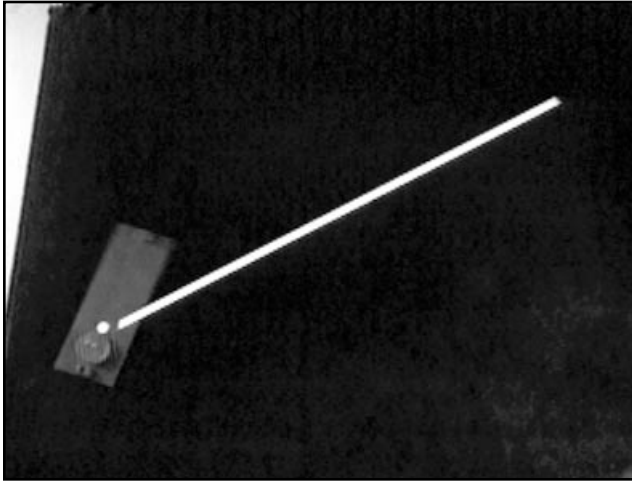
Figure 1

We'll use this air table to demonstrate the motion of an object's center of mass.

A flat piece of acrylic has been weighted on one side so that the center of mass is located beneath this orange spot.

To show that the center of mass is located at the spot, we can balance it on a finger at that point.

Now the acrylic is placed on the air table and given a push. The acrylic and the weight move in circles, but the center of mass moves in straight lines.

Equipment

1. Level air table.
2. Blower system.
3. Rectangular flat glider with an off-center attached weight, plus bumpers.
4. Mark center of mass of system to aid visibility.