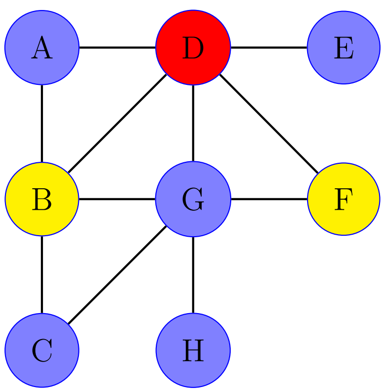
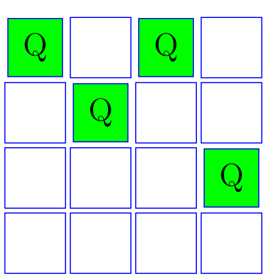
Local Search Exercises

Exercises

1. Let G be the simple graph shown below. The problem is to find a coloring of V using colors **red**,**blue**, and **yellow**, so that no two adjacent vertices are assigned the same color. If we model the problem with the set of variables $x_a, x_b, \ldots, x_g$, where, e.g., x_a denotes the color assigned to vertex a, Then define the state space associated with this model. How big is this space? Give an example of a solution state. For an arbitrary state s, define a "reasonable" neighborhood function $\nu(s)$ for s. Using these neighborhoods, provide a local path from the coloring shown below to your aforementioned solution state.



2. Repeat the previous problem for the 4-Queens problem. Assume the model variables are $x_1, \ldots, x_4$, where x_i indicates the row for which the queen in column i is placed. Define the state space associated with this model. How big is this space? Give an example of a solution state. For an arbitrary state s, define a "reasonable" neighborhood function $\nu(s)$ for s. Using these neighborhoods, provide a local path from the placement shown below to your aforementioned solution state.



3. A news station classifies each day as "good", "fair", or "poor" based on its daily ratings which fluctuates with what is occurring in the news. Moreover, the following table shows the probabilistic relationship between the type of news day of the current day and the probability of the type of news day of the next day conditioned on the type of current day.

current \ next	good	fair	poor
good	0.60	0.30	0.10
fair	0.50	0.25	0.25
poor	0.20	0.40	0.40

In the long run, what percentage of news days will be classified as "good"? Show work and explain.

4. Using the transition matrix from Exercise 3, determine the probability that the news day will be poor exactly three days after a good news day.

5. Consider a Markov chain with state space $\{1, 2, 3, 4, 5\}$ and transition matrix

$$\begin{bmatrix} 0.25 & 0.50 & 0.25 & 0 & 0 \\ 0.50 & 0.50 & 0 & 0 & 0 \\ 0 & 0 & 0.50 & 0.50 & 0 \\ 0 & 0 & 0.25 & 0.50 & 0.25 \\ 0 & 0 & 0 & 0 & 1.00 \end{bmatrix}.$$

Is the state graph for this chain connected? undirected? self-transitioning? Explain.

6. Verify that the $n \times n$ Markov-chain matrix with entries $P_{ij} = 1/n$, for all $1 \leq i \leq j \leq n$, has stationary distribution $\pi = (1/n, \ldots, 1/n)$. Hint: use the detailed-balance equations.

7. Consider the state space $S = \{00, 01, 10, 11\}$ of binary strings having length 2. Let $p(j|i) = 0.5$ if j differs from i in exactly one bit, and $p(j|i) = 0$ otherwise. Provide the transition matrix for this distribution. Suppose we desire an equilibrium distribution $\pi = (1/8, 1/8, 1/4, 1/2)$. For example, in the long run a random walk should be in state $3 = (1, 1)$ half the time. Provide the $\alpha(i, j)$ matrix that will be used if we are to obtain the desired equilibrium distribution using the Hastings-Metropolis algorithm. Provide the transition matrix that arises from the HM algorithm.

8. Consider the state space $X = \{0, 1\}^n$ of binary strings having length n. Let $p(y|x) = 1/n$ if y differs from x in exactly one bit, and $p(y|x) = 0$ otherwise. Suppose we desire an equilibrium distribution π for which $\pi(x)$ is proportional to the number of ones that occur in vector x. For example, in the long run a random walk should visit a string having five 1's five times as often as it visits a string having only a single 1. Provide a general formula for $\alpha(x, y)$ that would be used if we were to obtain the desired equilibrium distribution using the Hastings-Metropolis algorithm.