A^* Search Exercises

Exercises

1. Give an example that shows that not every admissible heuristic is monotone.

2. A robot is standing due north on square $(1, 1)$ of an $N \times N$ grid. The goal of the robot is to reach square (N, N). The robot has three possible actions: turn left, turn right, and move forward (either north, south, east, or west) to an adjacent square. However, throughout the grid there are walls that prevent the robot from moving forward from one square to an adjacent one (there are also walls around the entire grid boundary). Specify the state space for this problem, the goal-testing function, and the set of actions. What is the size of the state space? Provide a bound on the branching factor b, and a bound on the minimum goal depth d.

3. A farmer needs to cross a river with three of his possessions: a lion, goat, and cabbage. The boat can only hold the farmer, along with one possession. Thus, the farmer must make multiple trips across the river. Also, he must neither leave the lion alone with the goat, nor leave the goat alone with the cabbage. Provide a sequence of river crossings for the farmer so that all of his possessions can be safely transported across the river. Provide a Clara model for i) the set of legal states, and ii) the problem state graph. Include an implementation of the following functions.

   ```
   Boolean is_initial(State: s)
   Boolean is_goal(State: s)
   Set(State) next(State: s) //returns set of states one can transition to from s
   ```

   Hint: for the `next` function, is ok for a state in the returned set to not be legal.

4. For the 4-puzzle, show that the state space can be divided into two sets of equal size, such that no state from one set is adjacent to a state from the other set. Note: this property also holds for the 8-puzzle, and shows that in general not all states from a space are reachable from an initial state.

5. Given three jugs, one of 12 gallons, 8 gallons, and 3 gallons, and a water faucet, you are allowed to fill the jugs and/or empty them into one another, or on to the ground. The goal is to measure out exactly one gallon. Hint: every action must have the effect of either entirely emptying or filling one of the jugs. For example, if you pour from the 8-gallon jug, then it must either empty, or it must fill at least one other jug.

6. Consider the state space consisting of the numbers 1 to 15. The two possible actions on a state i are to multiply it by 2, or multiply it by 2 and add 1. Assuming 1 is the initial state, and 11 is the goal state. Show the order of the search for breadth-first, bounded depth-first (with depth bound of 3), Assume the $2k$ action has precedence over the $2k + 1$ action.

7. n checkers occupy squares $(1, 1)$ through $(1, n)$ of an $n \times n$ grid (i.e. they are at the bottom row of the grid). The checkers must be moved to the top row, but in reverse order. In other words, checker i must be moved to square $(n - i + 1, n)$. On each step each checker can move one square up, down, left, right, or stay put. If a checker does not move, then it may be jumped (vertically or horizontally) by one other checker that is adjacent to it. Two checkers cannot occupy the same square. Provide a nontrivial admissible heuristic h_i that estimates the number of steps needed for the i th checker to reach its goal square $(n - i + 1, n)$. Will $\sum_{i=1}^{n} h_i(n)$ be admissible? What about $\min_i(h_i(n))$? $\max_i(h_i(n))$?

8. In what way can breadth-first search be considered a special case of best-first search? Explain.

9. Prove that the set of nodes considered during an A* search is a subset of nodes considered during a breadth-first search.