STAT450/550 Multivariate Stat Analysis
Sample Midterm II   NAME_____

Write your answers on separate papers (single sided). Attach this test on the top of your papers.

1. Consider the following correlation matrix for a three-variate data of size 20.

$$\rho = \begin{bmatrix} 1 & & \\ .58 & 1 & \\ .35 & .61 & 1 \end{bmatrix}.$$

The eigenvalues of the matrix is given as 2.03, 0.65, and 0.31 with the corresponding eigenvectors of (.54, .63, .56), (-.73, .03, .68), and (.41, -.78, .48), respectively.
   (a) Write the estimated loading matrix for a factor analysis model with two common factors using the principal factor method
   (b) For the loading matrix in (b) provide the matrix of reproduced correlation matrix. Is the reproduced matrix close to the original correlation matrix above?
   (c) Calculate the communalities and specific variances of each of the variables.

2. Fifty bars of soap are manufactured in each of two ways. Two characteristics, X_1=lather and X_2=mildness, are measured. The summary statistics for bars produced by method 1 and 2 are

$$\bar{x}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \ S_1 = \begin{pmatrix} 2 & 1 \\ 1 & 5 \end{pmatrix} \text{ and } \bar{x}_2 = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \ S_2 = \begin{pmatrix} 3 & 1 \\ 1 & 4 \end{pmatrix}$$

   (a) Assuming equal covariance matrices, give the discriminant function rule and classify a new observation $x_0 = (2, 3)$ using the rule.
   (b) Now, assume that it is twice as costly to classify method1 to method2 than method2 to method1. Repeat (a). Note that we assume the equal priors.
   (c) Repeat (a) assuming unequal covariance matrices.