

STAT 580 : Applied Time Series Analysis
Midterm II exam

1. Suppose we have the simple regression model

$$y_t = \beta z_t + x_t,$$

where

$$x_t = \phi_1 x_{t-1} + \phi_2 x_{t-2} + w_t$$

$t=1,2,\dots,T$, where w_t is white noises with mean zero and variance σ_w^2 . Give the detailed Cochran-Orcutt procedure to estimate the β , ϕ_1 , ϕ_2 , and σ_w^2 .

2. Consider the first order seasonal moving average process

$$x_t = w_t - \Theta w_{t-2}.$$

- (a) Provide the inverted representation, $w_t = \Pi(B)x_t = \sum_{k=0}^{\infty} \pi_k x_{t-k}$. Note the initial condition $\pi_0 = 1$.
- (b) Determine the l -step forecasting function and its approximation.
- (c) Determine the l -step forecast variance.
3. Consider the AR(2) series $x_t = \phi x_{t-1} + \phi^2 x_{t-2} + w_t$, where $w_t \sim WN(0, \sigma_w^2)$.
- (a) Give values of ϕ to verify the invertibility condition.
- (b) Given $R_x^T(0) = 6.0$ and $\rho_x^T(1) = 0.7$ find estimates of ϕ and σ_w^2 by solving the Yule-Walker equations.