Department of Mathematics and Statistics, California State University, Long Beach

STAT 580 : Time Series Analysis, Fall 2011
Midterm exam I : Oct. 20

Note: To receive full credits you have to show all your work in details; Do not expect that I will understand any missing explanations or steps. You may use one page of note and no other material will be allowed.

1. Suppose that
$$x_t = \beta_0 + \beta_1 t + \beta_2 t^2 + w_t,$$
where β_0, β_1, and β_2 are constant and w_t is white noise with mean zero and variance σ_w^2.
   (a) Show that x_t is nonstationary series
   (b) Show that the second differecne $\triangle^2 x_t$ is a stationary series.

2. Suppose that
$$x_t = \sum_{j=0}^{m} w_{t-j}/(m+1).$$
   (a) Show that the series x_t is stationary.
   (b) Give the ACF of this process. Briefly discuss the ACF.
   (c) Give the power spectrum of the process.

3. Consider two time seires
$$x_t = \frac{1}{2}(w_t - w_{t-1})$$
$$y_t = \frac{1}{2}(w_t + w_{t-1})$$
where the white noise series w_t has mean zero and variance one.
   (a) Show that x_t and y_t are jointly stationary. Note in addition to autocovariances the cross covariance function must also be a function only of the lag h.
   (b) Express power spectra $f_x(\nu)$ and $f_y(\nu)$ in terms of $\cos(2\pi\nu)$.
   (c) Express cross spectrum $f_{yx}(\nu)$ and squared coherence $\rho_{yx}^2(\nu)$in terms of $\cos(2\pi\nu)$.

Spectral representation

$$R_x(h) = \int_{-1/2}^{1/2} f_x(\nu) e^{2\pi i \nu h} d\nu$$

and

$$f_x(\nu) = \sum_{h=-\infty}^{\infty} R_x(h) e^{-2\pi i \nu h}$$

Trignometric identities

$$e^{ix} = \cos(x) + i\sin(x)$$
$$e^{-ix} = \cos(x) - i\sin(x)$$
$$\cos(x) = (e^{ix} + e^{-ix})/2$$
$$\sin(x) = (e^{ix} - e^{-ix})/2$$