Assignment 3 Math 456 Fall 2024
Deadline: 13 December

Reading

Chaos, Ch. 3

Exercises

Write in concise, clear *sentences* (incorporating symbolic notation and computations).

1) ***Chaos*: 3.1** (p. 140)
2) **4-periodic maps.** Let $f : [\alpha, \beta] \longrightarrow [\alpha, \beta]$ have an orbit of period four $\{a, b, c, d\}$ where
$$a < b < c < d.$$
   a) Enumerate *all* of the possible ways that f can act on $a,\ b,\ c,\ d$. For instance,
   $$f(a) = b,\ f(b) = c,\ f(c) = d, f(d) = a.$$
   b) In each case, show that f also has an orbit of period 2.
   (*Note*: Don't invoke Sharkovskii's Theoreom—this result is an ingredient in our proof of that result.)
   c) In the case
   $$f(a) = b,\ f(b) = c,\ f(c) = d, f(d) = a$$
   show that f has orbits of *every* period.
3) (Bonus) We saw that the logistic map
$$\ell_4(x) = 4\,x(1-x)$$
is conjugate to the tent map
$$t_4(x) = \begin{cases} 2\,x & x \in [0, \frac{1}{2}] \\ -2\,(x-1) & x \in [\frac{1}{2}, 1] \end{cases}.$$
With a succinct argument, show that the logistic map
$$\ell_3(x) = 3\,x(1-x)$$
is *not* C^1-conjugate—that is, by a C^1 map—to the tent map
$$t_3(x) = \begin{cases} \frac{3}{2}\,x & x \in [0, \frac{1}{2}] \\ -\frac{3}{2}\,(x-1) & x \in [\frac{1}{2}, 1] \end{cases}.$$
Might they be *topologically conjugate*—by a homeomorphism?