

Talks 2 Math 456 Fall 2024

1) **Maps that contract.** Suppose $f : [a, b] \rightarrow [a, b]$,

$$|f(x) - f(y)| < |x - y| \quad \text{for all } x, y \in [a, b] \text{ where } x \neq y,$$

and that f is continuous on $[a, b]$.

Prove the following.

- a) There's a unique fixed point p in $[a, b]$.
- b) p is attracting.
- c) The basin of attraction of p contains (a, b) .
- d) Discuss what can happen at a and b .

2) Let $f(x) = x^2$. Find all mobius transformations T such that

$$T \circ f \circ T^{-1} = f.$$

3) Show that the family of logistic maps

$$x \longrightarrow ax(1 - x)$$

is conjugate to the quadratic family

$$x \longrightarrow x^2 + c.$$

- 4) **Chaos:** 1.6 (p. 36)
- 5) **Chaos:** 1.13 (p. 37)
- 6) **Chaos:** 1.17 (p. 38)