## Assignment 2 Math 456 Fall 2024 Due: Friday 1 November

## Reading

Chaos, 1.4 - 1.8

## Exercises

Write in concise, clear *sentences* (incorporating symbolic notation and computations).

- 1) *Chaos*: 1.6 (p. 36)
- 2) **Chaos:** 1.11 (p. 37)
- 3) Suppose p is  $\ell$ -periodic (that is,  $\ell$  is the period of p) under f and that

$$f^k(p) = p.$$

Show that  $\ell$  divides k.

- 4) Suppose that p is fixed by and attracting under a continuous map f. Let  $B_0(p) = (a, b)$  be the immediate basin of attraction of p—the largest interval containing p in which the orbit of each element converges to p.
  - a) Show that  $f(B_0(p)) \subset B_0(p)$ . Can you also show that  $f(B_0(p)) = B_0(p)$ ? Does this result require continuity? That is, can there be a discontinuous map g with g(p) = p and p attracting, but  $f(B_0(p)) \not\subset B_0(p)$ ?
  - b) Show that  $f(\{a, b\}) \subset \{a, b\}$ . If  $f(\{a, b\}) \neq \{a, b\}$ , give a condition that's necessary for this outcome.