Sample topics for a project talk — Math 444

Bear in mind: The quality of a project is directly related to its having **substantive content** and a **well-defined**, **narrow focus**. The following are intended to be **suggestive**. Many topics need a narrowing of focus. Computer graphics and experiments might help to illuminate some of the ideas and results.

- Classify the finite subgroups of the group of isometries of the plane.
- The set of "motions" of a sphere is an infinite group called O_3—3×3 matrices where $AA^T = I$ and $\det A = \pm 1$. Classify the *finite* subgroups of O_3.

  Source: M. Senechal, *Finding the Finite Groups of Symmetries of the Sphere*, The American Mathematical Monthly, 97, No. 4 (Apr 1990), pp. 329-335.
- Find all groups of order 8 (up to isomorphism).
- Find all finite subgroups of the complex numbers under multiplication.
- Use mobius transformations to express the group of symmetries of the regular tetrahedron T. Find three special polynomials that are invariant under this group. That is, a polynomial $P(z)$ such that

  $$P(Az) = P(z) \quad \text{for all } A \in T.$$
- Describe and count the rotational symmetries of the icosahedron (dodecahedron). Call this the *icosahedral group* I. Find five tetrahedra—a set of four points—each of which is preserved by a different "tetrahedral subgroup" in I. Show that I permutes these five tetrahedra so that I is a normal subgroup of S_5.
- Using the six pairs of antipodal vertices, express the icosahedral group I in terms of a subgroup of S_6—as permutations of six things. Exhibit the correspondence between the permutations of five tetrahedra and six pairs of antipodal vertices. This shows that I can be mapped into S_6 in a way that's not straightforward—that is, by fixing one thing.
- Consider the group $G = \mathbf{Z}_2 \times \mathbf{Z}_2 \times \mathbf{Z}_2$. Describe the subgroups of G. Each non-identity element has order two. An automorphism permutes the seven elements of order two. How many automorphisms of G are there?
- The imaginary number i is a square root of -1 and so has order 4 under multiplication. Show that a noncommutative group Q of order 8 is generated by i and another square root j of -1, where $j \neq i^3$. Work out a group graph for Q. Find all the subgroups of Q and show that each one is normal.

  Source: Grossman and Magnus, *Groups and Their Graphs*.