Exam 1 Math 444 Fall 2024
Deadline: Friday, 8 Oct

- Write concisely and clearly. Submit your work to the Canvas drop box.
- You can work with a partner–submit one paper for the group.
- Papers written in *LaTeX* will receive a 5% bonus.
- *Ethic*: In completing this exam, you should not consult other sources—excepting the text and instructor. The work represented by what you (and your group) submit should be entirely your own.

1 Group multiplication tables

For each of the following cases, can the table be the multiplication table of a group? If not, give a reason. If so, show that it is.

a)

	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	d	c	a	b
d	c	d	b	a

b)

	a	b	c	d
a	a	b	c	d
b	b	d	a	c
c	c	a	d	b
d	d	c	b	a

c)

	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	c	b	c

d) *Note*: Here, e does not signify the identity element. It's just an abstract element.

	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	c	a	e	f	d
c	c	d	b	f	a	e
d	d	e	f	a	b	c
e	e	f	d	b	c	a
f	f	a	e	c	d	b

2 Groups of order two and three.

a) Prove that, up to isomorphism, there is only one group with two elements.

b) Prove that, up to isomorphism, there is only one group with three elements.

Suggestion: Work with multiplication tables.

3 Groups whose elements have order two

Suppose that G is a group in which every non-identity element has order two. Show that G is commutative.

4 Cyclic group properties

Consider $\mathbf{Z}_n = \{0, 1, \ldots, n-1\}$ under addition mod n.

a) Show that an element k is a generator of $\mathbf{Z}_n$ if and only if k and n are relatively prime.

b) Is every subgroup of $\mathbf{Z}_n$ cyclic? If so, give a proof. If not, provide an example.