Exam 1 Math 362 Spring 2025 Due: 7 March

- Write concisely and clearly. Submit your work to the Canvas drop box.
- You can work with a partner–submit one paper for the group.
- Papers written in *LaTeX* will receive a 5% bonus.
- *Ethic*: In completing this paper you should not consult other sources—excepting the text and instructor. The work represented by what you write and submit should be entirely your own.

1 $\sin z = 0$

Recall that

$$\sin z = \frac{e^{iz} - e^{-iz}}{2\,i}.$$

a) Use the geometric description of $\sin z$ (treated in class) to show that the only solutions to $\sin z = 0$ are real.

b) By taking $z = x + iy$ and considering the exponential form for $\sin z$, verify analytically that the only solutions to $\sin z = 0$ are real.

2 Summing the roots of unity

A solution to the equation

$$z^n = 1$$

is called an nth root of unity.

a) What are all of the square, cube, and 4th roots of unity? Plot the respective roots of unity in the plane of complex numbers. What happens when you sum *all* of the square roots of unity? The cube roots of unity? The 4th roots of unity? Does the result makes sense geometrically?

b) Describe the nth roots of unity in polar form. On the basis of the above cases, form a conjecture on the nth roots of unity. Appealing to the geometric nature of complex multiplication, give an argument for you conjecture.

c) Work with the defining equation for the nth roots of unity to establish your conjecture algebraically.

3 Describing reflections

We know that the conjugation map

$$z \longrightarrow \bar{z}$$

reflects $\mathbf{C}$ across the real axis.

a) Describe the map

$$z \longrightarrow g_1(z)$$

that reflects $\mathbf{C}$ across an *arbitrary line through* 0. (What would be a good parameter(s) to use in describing such a line?)

Suggestion: Express the reflection as a combination of a suitable mobius transformation with the conjugation map. Make a diagram of the mapping process.

b) Now find the map

$$z \longrightarrow g_2(z)$$

that reflects **C** across an *arbitrary line*. (Again, what would be a good parameter(s) to use in describing such a line?)

c) Compute the composite map

$$z \longrightarrow g(f(z))$$

where the f and g are reflections through parallel lines. Is this a familiar transformation? (Choose convenient parameters.)

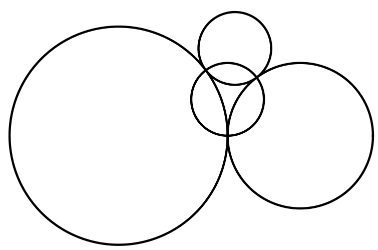
d) Compute the composite map

$$z \longrightarrow g(f(z))$$

where f and g are reflections through intersecting lines. Is this a familiar transformation? (Choose convenient parameters.)

4 Three tangent circles

Consider three circles C_1, C_2, C_3 that are mutually tangent—each circle is outside and tangent to the other two.



The three points of tangency determine a fourth circle that's perpendicular to each C_k. To show this, use a judiciously chosen mobius transformation to map the configuration to one that's particularly easy to understand. In light of what happens in the transformed situation, solve the original problem.

Does it matter whether the three tangent circles are exterior to one another? Why or why not?