

Exam items will be selected from assigned exercises not on WebAssign and the following.

- 1) Calculate the work done by the force field

$$\mathbf{F} = \langle x, y, z \rangle$$

on an object that's moved along the conical helix $\langle t \cos t, t \sin t, t \rangle$ from $t = 0$ to $t = 2\pi$.

Does the work done depend on the path along which the object is displaced in moving from its starting point to stopping point? Explain briefly. If the work done is path independent, determine a potential function P for \mathbf{F} and verify that the quantity of work in terms of P is the same as you found previously.

- 2) For two positive numbers x and y , the *arithmetic average* is $\frac{x+y}{2}$ and the *geometric average* is \sqrt{xy} . Consider the surfaces given by

$$z = a(x, y) = \frac{x+y}{2} \quad z = g(x, y) = \sqrt{xy} \quad \text{for } x \geq 0, y \geq 0.$$

- Sketch the graph of $g(x, y)$ for $x \geq 0$ and $y \geq 0$.
- Show that the tangent plane to $z = g(x, y)$ at any point $(x_0, y_0, z_0) \neq (0, 0, 0)$ passes through $(0, 0, 0)$.
- Find the intersection of $z = a(x, y)$ and $z = g(x, y)$. At each point in the intersection of the two surfaces, find 1) the tangent plane to $z = g(x, y)$ and 2) the direction in which $g(x, y)$ is increasing most rapidly.
- Use the geometry of the surfaces $z = a(x, y)$ and $z = g(x, y)$ to show that the arithmetic average of two numbers is always greater than or equal to the geometric average. That is, show that

$$a(x, y) \geq g(x, y).$$

When are the two averages equal?

- You want to build a water tank in the shape of a rectangular box that's open on one side and holds 10 cubic meters. Find the length, width, and height of a box that uses the *least* amount of material. Is there just one shape that works?
 - Build a box-shaped tank that's open on one side and uses 10 square meters of material. Find the length, width, and height of the box that holds the *most* water. Is there just one shape that works? How does the solution to this problem compare to the solution to the previous problem?
- In rectangular coordinates, the surface area A of the piece of the graph of $z = f(x, y)$ over a region R is given by

$$A = \iint_R \sqrt{f_x^2 + f_y^2 + 1} \, dx \, dy.$$

If we think of the function f in cylindrical coordinates, it becomes

$$z = F(\theta, r) = f(r \cos \theta, r \sin \theta).$$

Consider the function

$$z = g(x, y) = x^2 + y^2.$$

Express g in cylindrical coordinates:

$$z = G(\theta, r) = g(x(\theta, r), y(\theta, r)) = ??$$

- a) Find the *element of area* on the surface S defined by

$$z = G(\theta, r).$$

Suggestion: A sketch would be a good idea. In terms of r and θ , express two independent vectors that are tangent to S at a given point.

- b) Use an integral to express the surface area of S that's over the circle of radius 2 and centered at $(0,0)$ in the (r, θ) plane. Evaluate the integral. Sketch the piece of the surface the area of which you're finding.
- 5) Consider a tetrahedron (four-faced pyramid) T determined by the four points

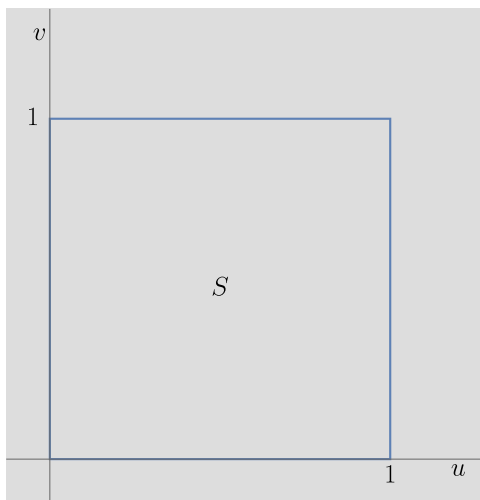
$$(0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c)$$

where a, b, c are all greater than 0.

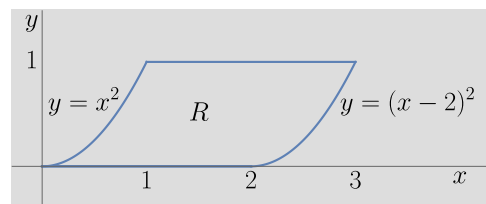
- a) Set up and evaluate an integral that gives the volume of T as a function of a, b, c .
- b) For a uniform (constant) density, find the center of mass of T as a function of a, b, c .
- c) Suppose the volume of T is required to be 9 cubic meters. Describe how to find the specific values of a, b, c that *minimize* the surface area of T . That is, set up the problem and explain what you would do, but don't do the calculations.
- d) **Bonus:** Do the calculations and find the specific a, b, c that minimize the surface area.
- 6) For the region R shown below, consider the integral

$$\iint_R (x - y) \, dA$$

- a) Work the integral directly by integrating over R .
- b) Work the integral by making a change of coordinates $(x, y) = T((u, v))$ and then integrating over S .



$$\xrightarrow{(x,y)=T(u,v)}$$



Answers:

1) $4\pi^2$

3a) $2a \times 2a \times a$ where $a = \left(\frac{5}{2}\right)^{1/3}$

3b) $2b \times 2b \times b$ where $b = \sqrt{\frac{5}{6}}$

4) b) $\frac{\pi}{6} (17\sqrt{17} - 1)$

5a) $\frac{abc}{6}$

5b) $\left(\frac{a}{4}, \frac{b}{4}, \frac{c}{4}\right)$

6) $\frac{7}{3}$