Study guide, midterm exam **Math 122** **Spring 2014**

Exam items will be similar to the following.

1) Imagine you ride your bike over three differing sections of trail and road. The first section takes 1.7 minutes, where you average 7 miles per hour. The second section takes 15 minutes, and you average 27 miles per hour. On the third section, you spend 14 minutes with an average speed of 40 miles per hour. What's your total distance traveled? What's your average speed for the entire trip?

2) Fill in the table of data below for average speed over the *preceding* time interval (from 0 to .1, .1 to .2, etc.). Plot the points for position against time. Sketch a graph of a *continuous* velocity function $v(t)$ that is consistent with the table values for average velocities. Make the velocity function as *simple* as you can. Describe the acceleration associated with $v(t)$.

   Briefly describe how you would go about estimating the instantaneous velocity when $t = .2$.

t (time)	x (position)	v (average speed)
0 s	0 m	$0 \ \frac{\text{m}}{\text{s}}$
.1	.5	?
.2	1.2	?
.3	2.2	?
.4	1.8	?

3)
   a) Explain in words and in symbols what it means to say that $g(x) = \sqrt{x}$ (where $\sqrt{x} \geq 0$) is the inverse of $f(x) = x^2$.
   b) What condition on x must be satisfied in order for $g(x) = f^{-1}(x)$?
   c) Can we select $h(x) = -\sqrt{x}$ to be the inverse of $f(x) = x^2$? If not, is $h(x)$ the inverse of *some* function? If so, what's $h^{-1}(x)$?

4) A function $f(x)$ that's defined on an interval $[a, b]$ has the *intermediate value property* (IVP) if it takes on *all* values between $f(a)$ and $f(b)$. That is, given *any* value y between $f(a)$ and $f(b)$, there is some x in $[a, b]$ where

$$f(x) = y.$$

   a) The *Intermediate Value Theorem* states that if a function is continuous on an interval $[a, b]$, then it has the IVP on $[a, b]$. Use a simple diagram to illustrate what this theorem says.
   b) Must a function that has the IVP over an interval $[a, b]$ be continuous over $[a, b]$? Why or why not? (Suggestion: Consider a graph.)

5) You decide to make an overnight hike up to the summit of Mount Baldy. For the trail you've chosen, it's a twelve-mile hike one way. The itinerary of your trip follows.

Saturday	8:00 AM	Leave the trailhead, hike at a steady pace
	12:00	After seven miles, stop for lunch
	1:00	Resume ascent at a steady pace
	6:00 PM	Arrive at the summit, camp overnight
Sunday	10:00 AM	Leave summit, returning on the same trail; steady pace
	1:00	After four miles., stop for lunch
	1:30	Resume descent at a steady pace
	5:00 PM	Arrive at the trailhead

a) Let $a(t)$ and $d(t)$ express your *distance along the trail* as a function of *time of day* during your ascent and descent respectively. On the same set of axes, sketch plausible graphs of $a(t)$ and $d(t)$.

b) On the ascent, if you slow down when the trail becomes more steep, is the trail steeper on the first part (before lunch) or the second?

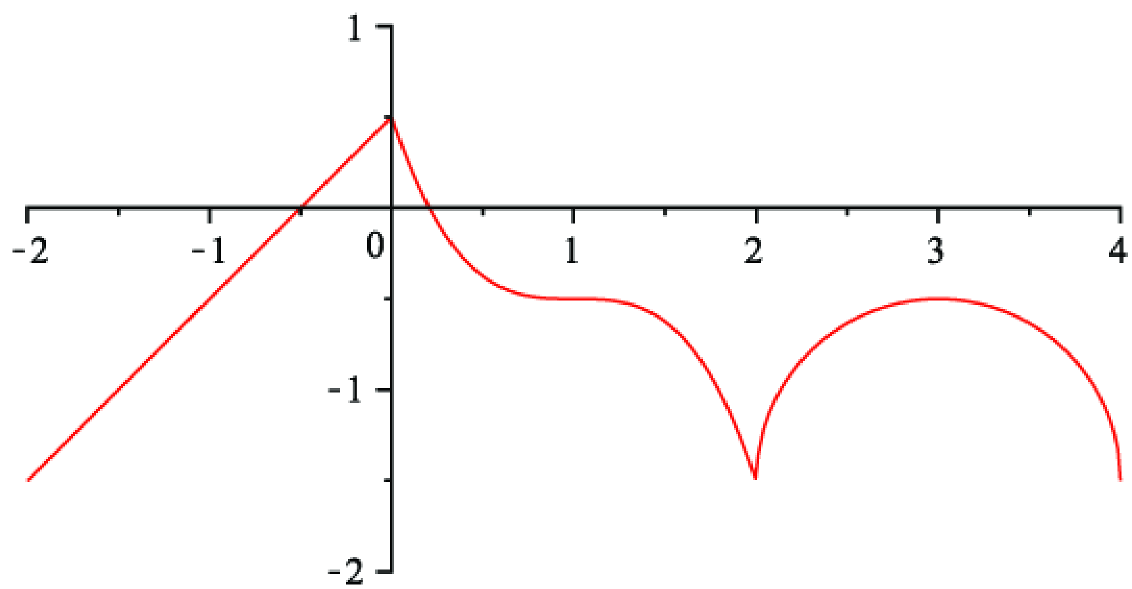
c) By considering the function

$$f(t) = a(t) - d(t)$$

explain why $f(t)$ has the intermediate value property and use the Intermediate Value Theorem to show that, on each day, there was a time of day when you were at the *same point on the trail.*

d) Estimate the time of day when you were at the same point.

6) The graph of a function is sketched in the following diagram. Graph the **first and second derivative** of the function. Make the sketch on the diagram and clearly indicate which graph is which derivative. Try to make it fairly accurate. Briefly explain how you're going about making the sketch.



7) From the economist Paul Krugman's blog:

> The economy's growth—the *rate of change* in GDP—depends on the *rate of change* in spending, not its level [the amount of spending]—and the rate of change [in spending] has been falling.

Note: GDP is Gross Domestic Product—a measure of the U.S. economy's output.

Interpret what Krugman is saying in the language of derivatives. Be precise and concise.

8) Find the following limits or state that and explain briefly why the limit doesn't exist. Sketch the graph of the function *near* the point that the independent variable approaches.

a) $\lim_{x\to 5}(x+3)$

b) $\lim_{x\to 5}\frac{x^2-2x-15}{x-5}$

c) $\lim_{u\to 0}\frac{|u|-1}{u^2-1}$

d) $\lim_{x\to 1}\frac{f(x)}{x-1}$ where $f(x)=\begin{cases}0 & x<1\\ 1 & x>1\end{cases}$

e) $\lim_{\theta\to 0}\frac{\sin\theta}{\cos\theta-1}$ *Suggestion*: Recall the identity $\sin^2\theta+\cos^2\theta=1$.

f) $\lim_{t\to\infty}\frac{\sin t}{t-1}$ Sketch the graph for large values of t.

9) Define the function $f(x)$ by

$$f(x)=\begin{cases}x^2 & x\geq 0\\ -x^2 & x<0\end{cases}.$$

a) Sketch the graph of $f(x)$.
b) Determine whether $f'(0)$ exists and if it does, what's the value?
c) Sketch the graph of $f'(x)$.
d) Determine whether $f''(0)$ exists and if it does, what's the value?
e) Sketch the graph of $f''(x)$.
f) By considering the concavity of the graph of $f(x)$, briefly explain what the value of $f''(0)$ would have to be if it exists.

10) The table below indicates the position relative to time of an object moving along a line.

a) Briefly discuss at which time T you would expect to be able to *best* estimate *instantaneous* velocity.
b) Carefully and concisely explain how to determine *exactly* the instantaneous velocity at $t=T$.
c) Estimate the instantaneous velocity at $t=T$.
d) Could your estimate for the instantaneous velocity be substantially *inaccurate*? If so, indicate how that could happen. If not, explain how it can't occur.
e) Is there a time when the velocity must be zero? If so, estimate when it occurs. If not, briefly explain why you can't tell.

t (seconds)	0	1	1.5	1.8	1.9	1.95	1.98	1.99	2
x (*meters*)	0	.7	1.3	2.2	1.8	1.7	1.62	1.59	1.58

11) Air is pumped into a spherical balloon at the constant rate of 3 liters per minute. Assume that when the pump starts the volume of the balloon is 0.

When the balloon contains 10 liters, how much time has passed? How fast is the radius of the balloon—in terms of centimeters—changing when it contains 10 liters? (1 liter = 1000 cm^3)