MATH 555: KNOT THEORY, HOMEWORK 5

MANIFOLDS AND SURFACES

Due by 10 am, Friday, March 8th

Problems (to turn in).

(1) Prove that the unit circle in $\mathbb{R}^2$ is a closed 1-manifold.
(2) Prove that the unit 2-sphere in $\mathbb{R}^2$ is homeomorphic to the polyhedral surface constructed from the following eight metric triangles. (Hint: Munkres 26.6)
   $\triangle(1,0,0)(0,1,0)(0,0,1)$
   $\triangle(-1,0,0)(0,1,0)(0,0,1)$
   $\triangle(-1,0,0)(0,-1,0)(0,0,1)$
   $\triangle(1,0,0)(0,-1,0)(0,0,1)$
   $\triangle(1,0,0)(0,1,0)(0,0,-1)$
   $\triangle(-1,0,0)(0,1,0)(0,0,-1)$
   $\triangle(-1,0,0)(0,-1,0)(0,0,-1)$
   $\triangle(1,0,0)(0,-1,0)(0,0,-1)$
(3) Prove that any polyhedral surface that is homeomorphic to a disk has an Euler Characteristic of one. (Hint: Induct on the number of triangles.)
(4) Prove or find a counter example to each of the following statements. You may use the classifications theorems from class in your proofs.
   (a) The Euler Characteristic is a perfect invariant of connected non-orientable polyhedral surfaces with boundary.
   (b) The Euler Characteristic is a perfect invariant of closed connected orientable polyhedral surfaces.