

# Math 550A, Homework 9

## Separation Axioms and Urysohn's Lemma

Due in class on May 6th

### Exercises (to do on your own)

1. Prove that  $\mathbb{R}_K$  is  $T_2$  but not  $T_3$ . (Recall  $\mathbb{R}_K$  from §13. Hint:  $K$  is a closed set in the  $\mathbb{R}_K$  topology.)
2. Let  $X$  and  $X'$  denote a single set under two different topologies. Suppose  $X'$  is finer than  $X$ . If  $X$  is Hausdorff, must  $X'$  be Hausdorff? Answer the same question for “ $T_1$ ”, “regular” and “normal” in place of “Hausdorff.”
3. Prove that if  $X$  is regular, then every pair of distinct points in  $X$  have neighborhoods whose closures are disjoint.

### Problems (to be turned in)

1. Munkres, §31, exercise 6.
2. Munkres, §32, exercise 1. (Note this can fail if we don't assume the subspace is closed – see example 1 of this section.)
3. Prove the Urysohn Lemma directly for a metric space  $(X, d)$ . (Hint: begin by defining the distance  $d(x, A)$  between a point  $x$  and a set  $A$ , and showing  $d(x, A)$  is continuous as a function of  $x$ .)