Math 123: Taylor's Formula and Approximations

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Outline

Taylor's Formula

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + R_n(x)$$

Where $R_n(x)$ is the **error term of order n**.

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Theorem (Taylor's Theorem)

Given a Taylor Series $\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$, if there is a constant M such that $|f^{(n+1)}(t)| < M$ for all t between a and x, then $|R_n(x)| < M\frac{|x-a|^{n+1}}{(n+1)!}$

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Uses: Can show Taylor series converges if $|R_n(x)|$ goes to zero as n goes to infinity, Can get estimates for functions.

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Show that the Maclaurin series for $cos(x)$ converges to $cos(x)$ for all x using Taylor's Theorem.

Examples

1. Show that the Maclaurin series for $\frac{1}{1-x}$ converges to $\frac{1}{1-x}$ for all $x \in [-\frac{1}{2}, \frac{1}{2}]$ by finding a formula for $R_n(x)$.
2. Estimate the error for approximating e^x on $[\frac{-1}{2}, \frac{1}{2}]$ using $1 + x + \frac{x^2}{2} + \frac{x^3}{6}$.
3. Estimate the error for approximating $cos(x)$ on $[-2\pi, 2\pi]$ using $1 + \frac{-x^2}{2} + \frac{x^4}{24}$.