

# Math 123: First Order D.E.s and Slope Fields

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# Outline

- 1 First Order Differential Equations
- 2 Slope Fields

# Types of Differential equations

## Definition

A differential equation is any equation involving a function, its derivatives.

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If the  $n$ -th derivative is the largest derivative that appears in the differential equation, we say it is an  **$n$ th order** differential equation.

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**Example:** Solve the initial value problem  $\frac{dy}{dx} = \frac{xe^x}{\cos(y)}$  and  $y(0) = 0$

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**Example:** Find  $\lim_{x \rightarrow \infty} y(x)$  if  $y(x)$  is a solution to the IVP  $y' = (y - 1)(y - 3)$  and  $y(0) = 0$ .

# Slope Fields Using Dfield

Here we will be using the free internet software Dfield.

**Example:** Determine the limits as  $x$  goes to infinity for solutions to  $y' = (\frac{1}{2}y(5 - y))$  (A Verhulst Equation).

**Example:** Determine the initial values for which solutions to  $y' = x^2 + y^2 - 4$  are **always** increasing.