

Math 123: Series Convergence Tests II

Ryan Blair

CSULB

Thursday March 24, 2016

Limit Comparison Test

Theorem

Let $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ be positive series. If

$$\lim_{i \rightarrow \infty} \frac{a_i}{b_i} = C$$

where C is a finite positive constant, then either both $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ converge or both $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ diverge.

Limit Comparison Test

Theorem

Let $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ be positive series. If

$$\lim_{i \rightarrow \infty} \frac{a_i}{b_i} = C$$

where C is a finite positive constant, then either both $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ converge or both $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ diverge.

Determine if $\sum_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$ is convergent or divergent.

Limit Comparison Test

Theorem

Let $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ be positive series. If

$$\lim_{i \rightarrow \infty} \frac{a_i}{b_i} = C$$

where C is a finite positive constant, then either both $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ converge or both $\sum_{i=1}^{\infty} a_i$ and $\sum_{i=1}^{\infty} b_i$ diverge.

Determine if $\sum_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$ is convergent or divergent.

Determine if $\sum_{i=1}^{\infty} \frac{2i^2-1}{i^2 3^i}$ is convergent or divergent.

Alternating Sequences

An **alternating** series is of the form $\sum_{i=1}^{\infty} a_i$ where $a_i = (-1)^i b_i$ or $a_i = (-1)^{i+1} b_i$ where $b_i \geq 0$ for all i .

Theorem

(Alternating Series Test)

If the alternating series $\sum_{i=1}^{\infty} a_i$ satisfies

- 1 $b_{i+1} \leq b_i$ for all i
- 2 $\lim_{i \rightarrow \infty} b_i = 0$.

Then $\sum_{i=1}^{\infty} a_i$ converges.

Alternating Sequences

An **alternating** series is of the form $\sum_{i=1}^{\infty} a_i$ where $a_i = (-1)^i b_i$ or $a_i = (-1)^{i+1} b_i$ where $b_i \geq 0$ for all i .

Theorem

(Alternating Series Test)

If the alternating series $\sum_{i=1}^{\infty} a_i$ satisfies

- 1 $b_{i+1} \leq b_i$ for all i
- 2 $\lim_{i \rightarrow \infty} b_i = 0$.

Then $\sum_{i=1}^{\infty} a_i$ converges.

Determine the convergence or divergence of $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$.

Alternating Sequences

An **alternating** series is of the form $\sum_{i=1}^{\infty} a_i$ where $a_i = (-1)^i b_i$ or $a_i = (-1)^{i+1} b_i$ where $b_i \geq 0$ for all i .

Theorem

(Alternating Series Test)

If the alternating series $\sum_{i=1}^{\infty} a_i$ satisfies

- 1 $b_{i+1} \leq b_i$ for all i
- 2 $\lim_{i \rightarrow \infty} b_i = 0$.

Then $\sum_{i=1}^{\infty} a_i$ converges.

Determine the convergence or divergence of $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$.

Determine the convergence or divergence of $\sum_{i=1}^{\infty} (-1)^i \frac{n}{n+1}$.

Alternating Sequences

An **alternating** series is of the form $\sum_{i=1}^{\infty} a_i$ where $a_i = (-1)^i b_i$ or $a_i = (-1)^{i+1} b_i$ where $b_i \geq 0$ for all i .

Theorem

(Alternating Series Test)

If the alternating series $\sum_{i=1}^{\infty} a_i$ satisfies

- 1 $b_{i+1} \leq b_i$ for all i
- 2 $\lim_{i \rightarrow \infty} b_i = 0$.

Then $\sum_{i=1}^{\infty} a_i$ converges.

Determine the convergence or divergence of $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$.

Determine the convergence or divergence of $\sum_{i=1}^{\infty} (-1)^i \frac{n}{n+1}$.

Determine the convergence or divergence of $\sum_{i=1}^{\infty} \cos(n\pi) \frac{1}{n^{\frac{2}{3}}}$.

Conditional and Absolute Convergence

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **absolutely** convergent if $\sum_{i=1}^{\infty} |a_i|$ is convergent.

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **conditionally** convergent if $\sum_{i=1}^{\infty} |a_i|$ is divergent and $\sum_{i=1}^{\infty} a_i$ is convergent.

Conditional and Absolute Convergence

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **absolutely** convergent if $\sum_{i=1}^{\infty} |a_i|$ is convergent.

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **conditionally** convergent if $\sum_{i=1}^{\infty} |a_i|$ is divergent and $\sum_{i=1}^{\infty} a_i$ is convergent.

Determine conditional or absolute convergence of $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$.

Conditional and Absolute Convergence

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **absolutely** convergent if $\sum_{i=1}^{\infty} |a_i|$ is convergent.

Definition

A series $\sum_{i=1}^{\infty} a_i$ is **conditionally** convergent if $\sum_{i=1}^{\infty} |a_i|$ is divergent and $\sum_{i=1}^{\infty} a_i$ is convergent.

Determine conditional or absolute convergence of $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$.

Determine conditional or absolute convergence of $\sum_{i=1}^{\infty} \frac{\cos(i)}{i^2}$.

Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i \rightarrow \infty} \left| \frac{a_{i+1}}{a_i} \right| = L,$$

then

- 1 If $L < 1$, the series converges absolutely.
- 2 If $L = 1$, the test is inconclusive.
- 3 If $L > 1$, the series diverges.

Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i \rightarrow \infty} \left| \frac{a_{i+1}}{a_i} \right| = L,$$

then

- 1 If $L < 1$, the series converges absolutely.
- 2 If $L = 1$, the test is inconclusive.
- 3 If $L > 1$, the series diverges.

Does not help for the mundane: $\sum_{i=1}^{\infty} \frac{1}{i^2}$

Ratio Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i \rightarrow \infty} \left| \frac{a_{i+1}}{a_i} \right| = L,$$

then

- 1 If $L < 1$, the series converges absolutely.
- 2 If $L = 1$, the test is inconclusive.
- 3 If $L > 1$, the series diverges.

Does not help for the mundane: $\sum_{i=1}^{\infty} \frac{1}{i^2}$

Helps with the crazy stuff: $\sum_{i=1}^{\infty} \frac{i^i}{i!}$

Root Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i \rightarrow \infty} |a_i|^{\frac{1}{i}} = L,$$

then

- 1 If $L < 1$, the series converges absolutely.
- 2 If $L = 1$, the test is inconclusive.
- 3 If $L > 1$, the series diverges.

Root Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i \rightarrow \infty} |a_i|^{\frac{1}{i}} = L,$$

then

- 1 If $L < 1$, the series converges absolutely.
- 2 If $L = 1$, the test is inconclusive.
- 3 If $L > 1$, the series diverges.

Helps with series involving variable powers: $\sum_{i=1}^{\infty} \left(\frac{i}{3i+4}\right)^i$

Root Test

Theorem

Given a series $\sum_{i=1}^{\infty} a_i$. If

$$\lim_{i \rightarrow \infty} |a_i|^{\frac{1}{i}} = L,$$

then

- 1 If $L < 1$, the series converges absolutely.
- 2 If $L = 1$, the test is inconclusive.
- 3 If $L > 1$, the series diverges.

Helps with series involving variable powers: $\sum_{i=1}^{\infty} \left(\frac{i}{3i+4}\right)^i$

Helps with series involving variable powers: $\sum_{i=1}^{\infty} i\left(\frac{2}{3}\right)^i$