

# Math 123: Series

Ryan Blair

CSU Long Beach

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# Outline

## 1 Series

# Series in terms of Sequences

Roughly, an infinite series  $\sum_{i=1}^{\infty} a_i$  denotes the sum of the terms in the sequence  $\{a_i\}_{i=1}^{\infty}$ .

## Definition

The **n-th partial sum** for a sequence  $\{a_i\}_{i=1}^{\infty}$  is

$$S_n = \sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n$$

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**Exercise:** Given a constant  $r$  find  $\sum_{i=0}^{\infty} r^i$  when it exists.

**Exercise:** Use partial sums to find  $\sum_{i=1}^{\infty} \frac{1}{i^2+i}$ .

# First tests for convergence

## Theorem

*If a series  $\sum_{i=1}^{\infty} a_i$  converges then  $\lim_{n \rightarrow \infty} a_n = 0$ .*

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**Exercise:** Determine the convergence or divergence of  $\sum_{i=1}^{\infty} \frac{e^i}{i^2}$ .



# Integral Test

## Theorem

Let  $f$  be a continuous, positive, decreasing function on  $[c, \infty)$ . If  $a_i = f(i)$ , then

- 1 If  $\int_c^{\infty} f(x)dx$  is convergent, then  $\sum_{i=c}^{\infty} a_i$  is convergent.
- 2 If  $\int_c^{\infty} f(x)dx$  is divergent, then  $\sum_{i=c}^{\infty} a_i$  is divergent.

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Show that  $\sum_{i=1}^{\infty} \frac{1}{i^p}$  is convergent for  $p > 1$ .

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Show that  $\sum_{i=1}^\infty \frac{1}{i^p}$  is convergent for  $p > 1$ .

Determine if  $\sum_{i=2}^\infty \frac{1}{i \ln(i)}$  is convergent or divergent.

# The Direct Comparison Test

## Theorem

Let  $\sum_{i=1}^{\infty} a_i$  and  $\sum_{i=1}^{\infty} b_i$  be positive series.

- 1 If  $a_i \leq b_i$  for all  $i$  and  $\sum_{i=1}^{\infty} b_i$  converges, then  $\sum_{i=1}^{\infty} a_i$  converges.
- 2 If  $a_i \leq b_i$  for all  $i$  and  $\sum_{i=1}^{\infty} a_i$  diverges, then  $\sum_{i=1}^{\infty} b_i$  diverges.

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Determine if  $\sum_{i=2}^{\infty} \frac{1}{i!}$  is convergent or divergent.

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Determine if  $\sum_{i=2}^{\infty} \frac{1}{i!}$  is convergent or divergent.

Determine if  $\sum_{i=2}^{\infty} \frac{\sqrt{i}}{2i-1}$  is convergent or divergent.

# Limit Comparison Test

## Theorem

Let  $\sum_{i=1}^{\infty} a_i$  and  $\sum_{i=1}^{\infty} b_i$  be positive series. If

$$\lim_{i \rightarrow \infty} \frac{a_i}{b_i} = C$$

where  $C$  is a finite positive constant, then either both  $\sum_{i=1}^{\infty} a_i$  and  $\sum_{i=1}^{\infty} b_i$  converge or both  $\sum_{i=1}^{\infty} a_i$  and  $\sum_{i=1}^{\infty} b_i$  diverge.

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Determine if  $\sum_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$  is convergent or divergent.



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Determine if  $\sum_{i=1}^{\infty} \frac{i+2}{(i+1)^3}$  is convergent or divergent.

Determine if  $\sum_{i=1}^{\infty} \frac{2i^2-1}{i^2 3^i}$  is convergent or divergent.