

1. HOMEWORK 7

Due: In Lecture ??

Problem 1. (a) Check that these two definitions coincide.

(b) Suppose that (x_1, x_2, \dots, x_n) are local coordinates in a neighborhood of the point $x \in M$, and that the tangent vector V at x is given in these coordinates by

$$V = v^1 \frac{\partial}{\partial x^1} + \dots + v^n \frac{\partial}{\partial x^n}.$$

Show that the rate of change of f at x with respect to V is given by

$$\sum_i v^i \left(\frac{\partial f}{\partial x^i} \right) (x).$$

Problem 2. (a) Show that

$$L_v(W_1 + W_2) = L_v(W_1) + L_v(W_2).$$

(b) Show that

$$L_v(fW) = (L_v f)W + f(L_v W) = (Vf)W + f(L_v W).$$

Hint: Remember that the Leibniz Rule in Freshman Calculus was proved by adding and subtracting a convenient middle term.

Problem 3. 18-1 on page 491 of Lee

Problem 4. 18-5 on page 491 of Lee