## 1. Homework 7

**Due: In Lecture 10-7**

**Problem 1.** Show that with this choice of $a$, the one-to-one immersion $\pi \circ F : M^m \to \mathbb{R}^{2m+1}$ is proper, and hence, is an embedding. (**See lecture notes for context**)

**Problem 2.** Prove that every $k$-dimensional compact concrete manifold can be immersed in $\mathbb{R}^{2k}$

**Problem 3.** Let $S(M)$ be the set of points $(x, y)\epsilon TM$ with $|v| = 1$. Prove that $S(M)$ is a $2k-1$-dimensional submanifold of $TM$.

**Problem 4.**If $df_x$ is surjective, $f$ is called a submersion at $x$. A map that is a submersion at every point is simply called a submersion.

Prove that $p : TM \to M$ given by $p(x, v) = x$ is a submersion.