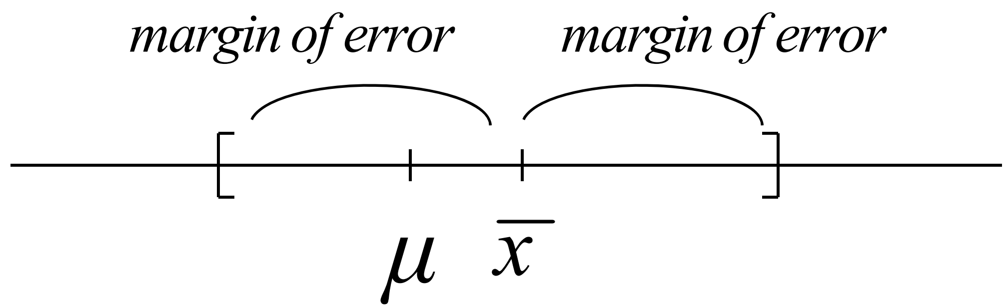
STAT 108 Lecture 14 Th, Oct 22, 2015

8.1 Confidence Interval

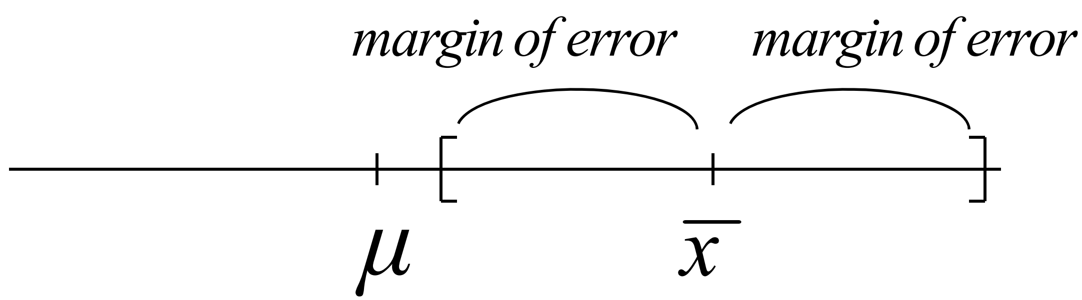
Definition. A $100\cdot(1-\alpha)\%$ **confidence interval (CI)** for a population parameter has the form

point estimate +/- margin of error

This interval covers the true population parameter $100\cdot(1-\alpha)\%$ of the time, and doesn't cover $100\cdot\alpha\%$ of the time.



$$covers\ 100\cdot(1-\alpha)\%\ of\ the\ time$$



$$doesn't\ cover\ 100\cdot\alpha\%\ of\ the\ time$$

Note that:

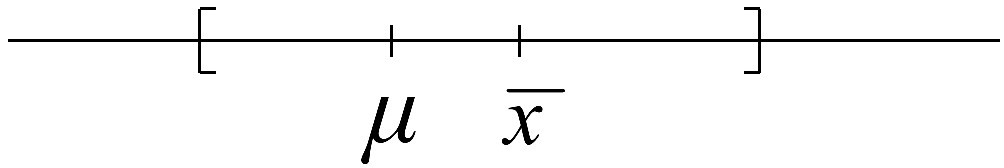
- A confidence interval is an interval estimator
- The center of a confidence interval is a point estimator
- The length of a confidence interval equals twice the margin of error

Definition. The quantity α is called the **confidence level**.

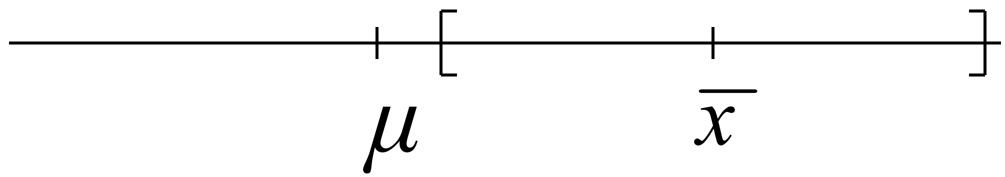
Typically three confidence levels are used: $\alpha = 0.1,\ 0.05$, or 0.01, which correspond to 90%, 95%, or 99% confidence intervals.

α	$100 \cdot (1-\alpha)\%$
0.1	100·(1 - 0.1)% = 90%
0.05	100·(1 - 0.0.5)% = 95%
0.01	100·(1 - 0.01)% = 99%

95% Confidence Interval

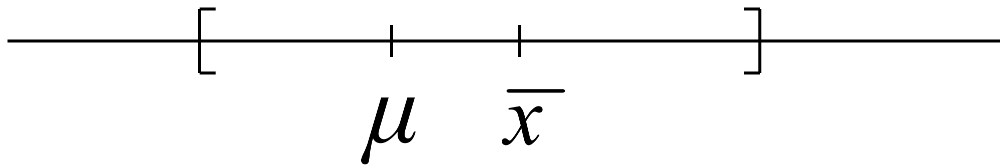


covers 95% *of the time*

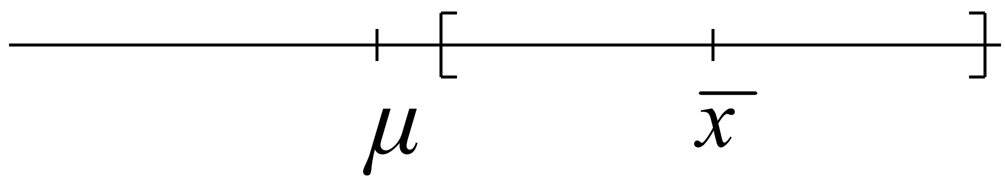


doesn't cover 5% of the time

99% Confidence Interval



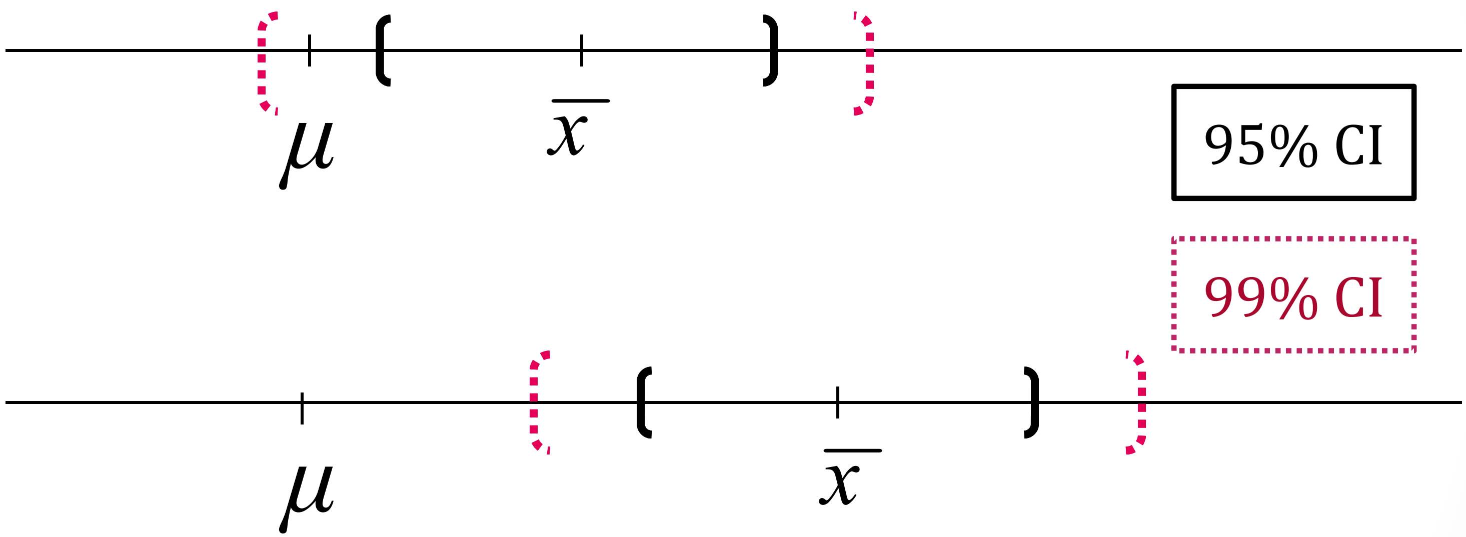
covers 99% *of the time*



doesn't cover 1% of the time

Question. For the same sample, which confidence interval is wider: 95% or 99% ?

Answer. 99% CI is wider , because it allows only 1% mistakes as opposed to 5% that 95% CI allows.

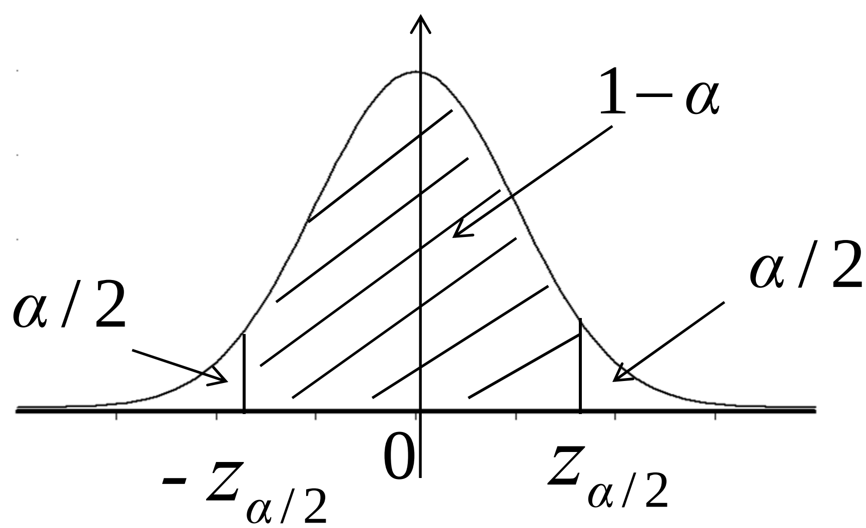


μ
$\bar{x}$
95% CI
99% CI
μ
$\bar{x}$

Question. If we increase the sample size, will the length of a CI increase or decrease?

Answer. If we sample the entire population, then the margin of error would be equal to zero, and so, the larger the sample size, the smaller the CI.

Definition. A **critical value** $z_{\alpha/2}$ that corresponds to a $100\cdot(1-\alpha)\%$ CI satisfies the equality $P(-z_{\alpha/2} \le z \le z_{\alpha/2}) = 1-\alpha$

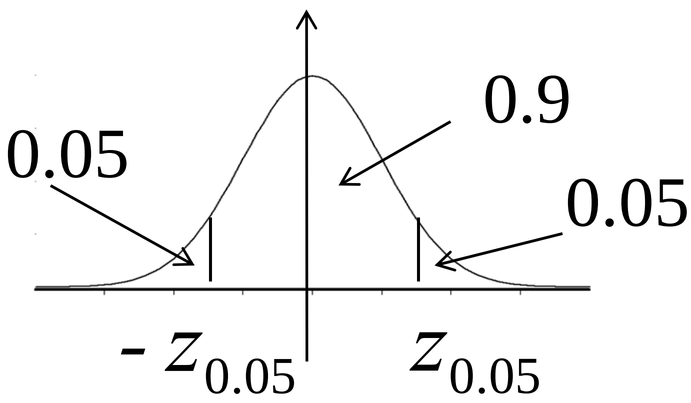


Example. Find the critical value for a 90% CI.

Solution. $\alpha = 0.1$, therefore, $\alpha / 2 = 0.05$

We want to find $z_{0.05}$ that satisfies

$P(-z_{0.05} \leq z \leq z_{0.05}) = 0.9$, or $P(z \leq -z_{0.05}) = 0.05$



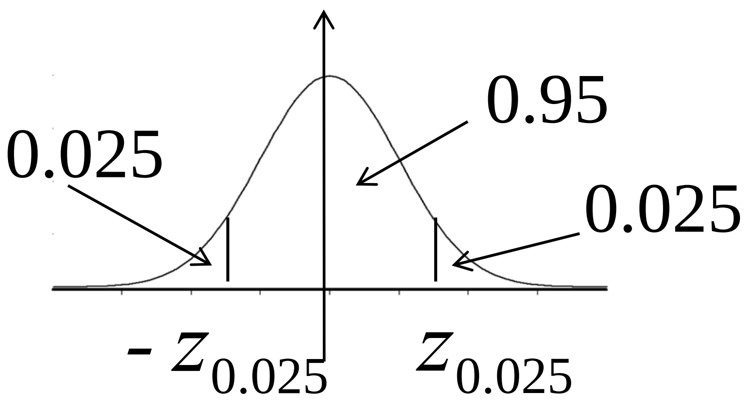
Thus, $\boxed{z_{0.05} = 1.645}$

Example. Find the critical value for a 95% CI.

Solution. $\alpha = 0.05$, therefore, $\alpha / 2 = 0.025$

We want to find $z_{0.025}$ that satisfies

$P(-z_{0.025} \le z \le z_{0.025}) = 0.95$, or $P(z \le -z_{0.025}) = 0.025$



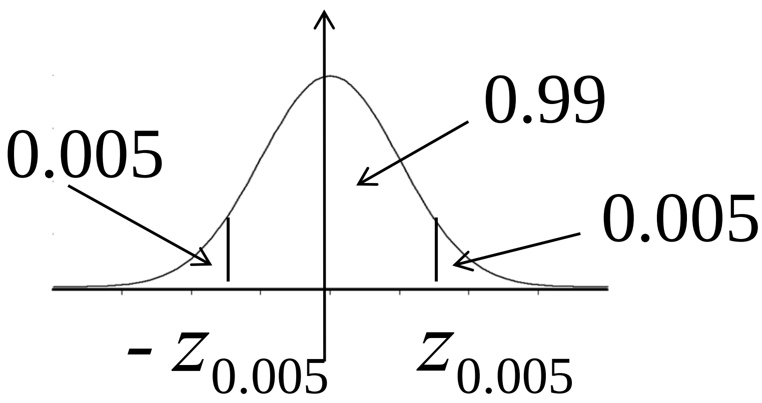
Thus, $\boxed{z_{0.025} = 1.96}$

Example. Find the critical value for a 99% CI.

Solution. $\alpha = 0.01$, therefore, $\alpha / 2 = 0.005$

We want to find $z_{0.005}$ that satisfies

$P(-z_{0.005} \le z \le z_{0.005}) = 0.99$, or $P(z \le -z_{0.005}) = 0.005$



Thus, $\boxed{z_{0.005} = 2.575}$