Applications of Systems of Linear Equations to Electrical Networks

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Introduction

Electrical networks are a specialized type of network providing information about power sources, such as batteries, and devices powered by these sources, such as light bulbs or motors. A power source forces a current to flow through the network, where it encounters various resistors, each which requires a certain amount of force to be applied in order for the current to flow through.

Systems of linear equations are used to determine the currents through various branches of electrical networks.

Known

Ohm's Law

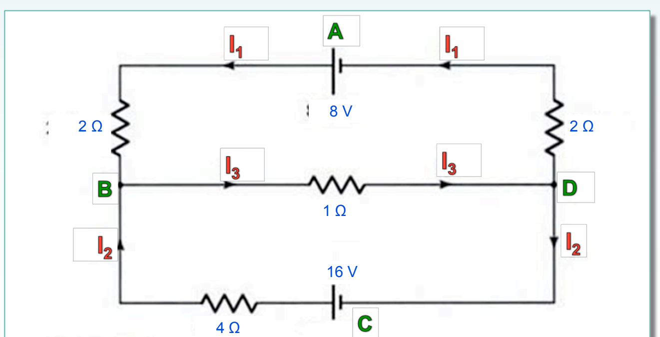
- The voltage drop across a resistor is given by V = IR

Kirchhoff's Law

- **Junction**: All the current flowing into a junction must flow out of it.
- **Path**: The sum of the IR terms in any direction around a closed path is equal to the total voltage in the path in that direction.

Methods

We wish to determine the currents I_1, I_2 and I_3 in the below circuit. Applying Ohm's and Kirchhoff's Law, we can construct a system of linear equations .



Results

Let the currents in the various branches of the circuit be I_1, I_2 and I_3 . Applying Kirchhoff's Law, there are two junctions in the circuit namely the points B and D. There are two closed paths ABDA and CBDC. Applying Kirchhoff's Law to the junctions and paths results in:

JUNCTIONS:

B: $I_1 + I_2 = I_3$

D: $I_3 = I_1 + I_2$

These two equations result in a single linear equation:

$$I_1 + I_2 - I_3 = 0$$

PATHS:

ABDA: $2I_1 + 1I_3 + 2I_1 = 8$

CBDC: $4I_2 + 1I_3 = 16$

We know have a system of three linear equations in three unknowns.

The problem thus reduces to solving the following system of three linear equations in three variables:

$$\begin{aligned} I_1 + I_2 - I_3 &= 0 \\ 4I_1 + 1I_3 &= 8 \\ 4I_2 + 1I_3 &= 16 \end{aligned}$$

This given system can be placed into an augmented matrix. We can now use row reduction algorithms to obtain an equivalent augmented matrix.

$$\begin{pmatrix} 1 & 1 & -1 & 0 \\ 4 & 0 & 1 & 8 \\ 0 & 4 & 1 & 16 \end{pmatrix} \underset{R2 + (-4)R1}{\sim} \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & -4 & 5 & 8 \\ 0 & 4 & 1 & 16 \end{pmatrix}$$

$$\underset{(-1/4)R2}{\sim} \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -5/4 & -2 \\ 0 & 4 & 1 & 16 \end{pmatrix} \underset{\substack{R1 + (-1)R2 \\ R3 + (-4)R2}}{\sim} \begin{pmatrix} 1 & 0 & 1/4 & 2 \\ 0 & 1 & -5/4 & -2 \\ 0 & 0 & 6 & 24 \end{pmatrix}$$

$$\underset{(1/6)R3}{\sim} \begin{pmatrix} 1 & 0 & 1/4 & 2 \\ 0 & 1 & -5/4 & -2 \\ 0 & 0 & 1 & 4 \end{pmatrix} \underset{\substack{R1 - (1/4)R3 \\ R2 + (5/4)R3}}{\sim} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

Summary

From the equivalent row reduced augmented matrix, it is apparent the system is consistent and thus a solution exists. The currents I_1, I_2 , I_3 are as follows:

I_1 = 1 Amp

I_2 = 3 Amps

I_3 = 4 Amps

The solution can be verified by substituting the values of the current into the original three linear equations with three unknowns.

Conclusion

The model for current flow is linear precisely because Ohm's Law and Kirchhoff's Law are linear: the voltage drop across a resistor is proportional to the current flowing through it (Ohm), and the sum of the voltage drop in a loop equals the sum of the voltage sources in the loop (Kirchhoff).

In practice, electrical networks can involve many resistances and circuits. Determining currents through branches involves solving large systems of equations that would require a computer.

References

Gareth, Williams,. Linear algebra with applications. Boston: Jones and Bartlett, 2004.