TAKE-HOME EXP. # 7a
Light Imagined as Traveling in Straight Lines: Sun Images in the Shade of a Tree.

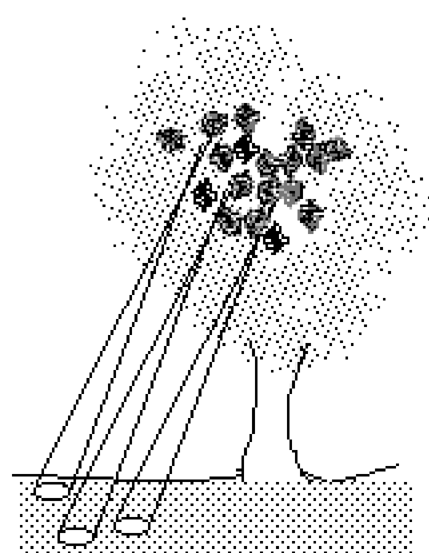
Our ordinary senses imply that light travels in a straight line. In fact, Isaac Newton was so convinced of light's straight-line travel that he proposed a particle model for light, saying:

> "Are not the rays of light very small bodies [particles] emitted from shining substances? For such bodies will pass through uniform mediums in straight lines without bending into the shadow,. . . "

This experiment examines phenomena that rely completely on explanations that assume straight-line travel for light. Optics studied with this assumption is called "geometric optics", because the drawings showing such behavior for light look like the figures one can find in a geometry book. Straight-line propagation of light energy appears so commonly in our experience that our brain assumes it is *always* the case—and will even produce the illusion of image located from where it thinks the light came from. For example, call to mind your own image *behind* plane mirrors, where no light from you is present!

Sun Images In The Shadow Of A Tree[*].

These observations again rely on straight-line propagation of light. That is, we will assume we can trace the progress of light by drawing a straight-line "ray". This experiment can lead to rough measurements of the diameter of the Sun, given its distance from Earth. To do so with a simple measurement is again a remarkable feat showing the power of good observations and some mathematics. *An approximate geometric model of straight-line light travel has proven to be enormously useful.*

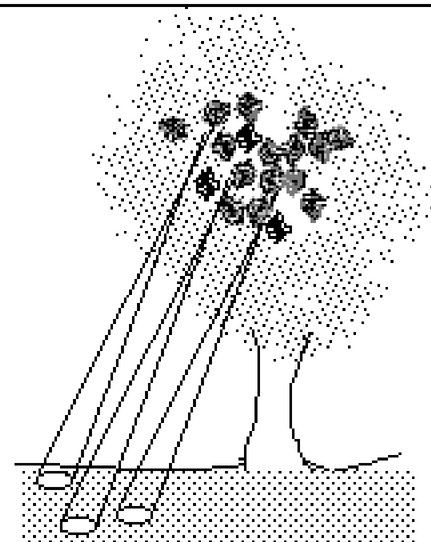


[*] Much of this writeup and experiment on sun images under trees is inspired by observations written up in a wonderful classic book by M. J. G. Minnaert called "Light and Color in the Outdoors". First published around 1937 in Dutch, it has just been newly translated and revised by Len Seymour and published in paperback by Springer-Verlag, 1996. An earlier inexpensive version was published by Dover in 1954 and may still be available, but it lacks photographs.

Any tree with relatively many small leaves will work. Many local areas produce very good images of the Sun. One area is the table area underneath the leafy trellises in front of the Nugget; take a look at the sun images on the tabletops. Another area is the tree near the southwest entrance of PH1. Another is the shade of the small tree outside the office of the Dean of the College of Natural Sciences, between PH1 and PH2. Another is a heavily shaded area between buildings PH2 and PH3, opposite the front of the Bookstore, although the Sun should be fairly high in the sky. Many trees and bushes can produce Sun images; just look carefully at the sidewalks.

HINT: Try these observations when the Sun is relatively high in the sky, perhaps 11 AM to 2 PM, although it works anytime. If the Sun is relatively low in the sky, as it is in the winter and earlier or later in the day, you need to make sure that your viewing screen is perpendicular to the rays of the Sun. If your shadow is more than half your height, I would consider the Sun to be relatively low in the sky.

Observation A : Look carefully at the lighted spots within the shade. *Nearly all of the lighted spots are composed of circles or ellipses of light! What's that you say? You don't see the circles of light?*

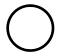
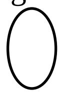


Please look more carefully using the following procedures.
You need a clear, sunny day.

1) Try to find a single small spot of light in the tree's shadow.

2) Use a piece of paper on a book as a screen for the light spot. *Tilt the screen until the plane of the screen is perpendicular to the direction of the incoming sunlight.* The more perpendicular the screen is to the incoming light, the more circular the image should become. Try it with a several more spots of light until you get one that is a single image, rather than overlapping images.

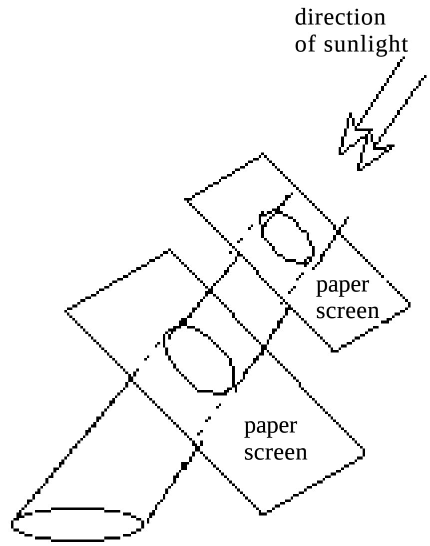
3) If you don't see the circles of light, try again under a different tree and/or at a different time of day. Take a partner with you to help. Don't give up till you see the circles! These circles are images of the Sun made by light coming through tiny "holes" in the canopy of leaves.

A question immediately arises: Shouldn't the light that comes through an opening show the shape of the opening? *How can the light on the ground be circles*  *or ellipses*  *?*

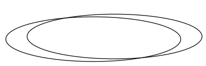
Observing carefully tells you directly that most of the spots of light on the ground under trees are areas of light formed by single images or several overlapping images of circles and ellipses.

As Marcel Minnaert says in his book, *Light and Color in the Outdoors,*

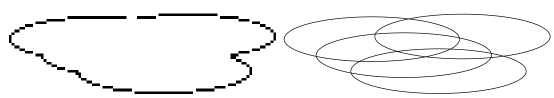
> "It is obviously impossible that all the holes and slits in the foliage happen to have the same shape. If you intercept . . . the rays on a piece of paper held at right angles to the sunlight, the image is no longer elliptical but round. Hold the piece of paper higher and higher and you will note that the spot of light gets smaller and smaller. This shows that the beam of light causing the spot is conical: the image is only elliptical because the ground cuts the cone at an angle."



If two small holes in the foliage are quite close together the images can overlap, leading to several identifiable kinds of spots.



An overlap of images from two holes very close together can produce what looks like a single ellipse, shown above, but it will be fuzzy and brighter than ellipses produced by a single hole.



An overlap of images from several nearby holes can produce what looks like an irregular blob of light, as shown above left, but is in fact produced by summing the light from 4, 5, or more ellipses, as shown to the right.

The Experiment: Producing Sun Images with a Pinhole and Reasoning to the Size of the Sun.

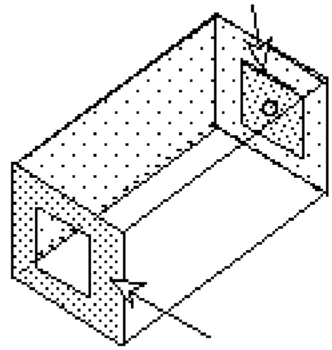
Determine the diameter of the Sun. On the face of it, it may seem an absurd task. However, all we need is a pinhole camera and geometry to measure the Sun.

"Pinhole cameras" have been known for thousands of years. Aristotle described such a device as a "dark chamber" with a small hole in the wall. The Latin words for "dark chamber" are "camera obscura", which is the name by which the device was known in the nineteenth century. They are often used to look at eclipses of the Sun, since looking at the Sun directly can cause serious eye damage.

CAUTION! Never stare at the Sun! There are no pain nerves in your retina, and the damage will be done before your feel anything. CAUTION!

Although the pinhole camera is described here in terms of construction instructions, *we are not asking you to build such a device, simply to understand it.*

Imagine an empty shoe box. Cut approximately a 10-centimeter square opening in each end of the box. One side will be the screen, the other side will have the pinhole. On the inside of the box, tape a piece of tissue paper that completely covers one of the openings. This tissue paper will act as a screen onto which we will project an image.



On the other side of the shoe box tape a piece of foil, like household aluminum foil, that completely covers the other opening that you cut.. Using a pin, carefully punch a small hole in the center of the foil.
You may have to experiment later by changing the size of the hole. This can be done by covering the original hole with opaque tape, and punching a new one. It doesn't have to be right at the center. Put the lid back on. Point the "hole end" of the box toward a brightly lighted scene or object (like a light bulb). Line up the length of the box so that light that goes through the hole will hit the screen.

To understand what is happening, follow the straight-line paths of rays of light travel from the object and through the pinhole of the camera. First , consider a light source that looks like an arrow. The arrow could represent a person or a tree, but an arrow is easy to draw and to specify it's orientation. A source of light is generally known as the "object" in an optics system. Imagine that the object-arrow emits light in all directions. How many straight-line paths from *the top of the arrow* will go through the region of the hole and hit the screen? We can say approximately just one! *All other light rays moving away from the top of the arrow at slightly different angles will not get through the hole* . Also, light rays from any other part of the arrow cannot get to the location where the ray from the top strikes the screen. Try drawing a ray from some other part of the object to hit the same location by drawing some lines from the object to the screen *through the hole*!

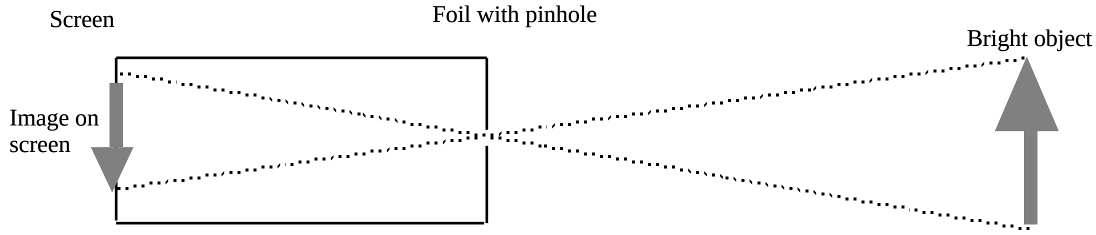
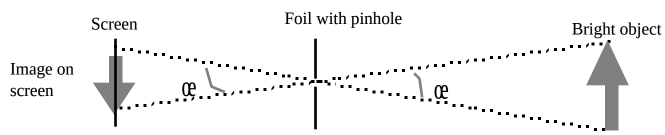


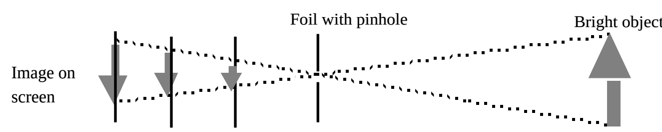
Image Orientation. Note that because of the pinhole's geometric constraint on which ray can hit the screen at which particular location, the screen will have an inverted (upside-down) image of the arrow. There will also be a right-to-left reversal around the vertical symmetry axis.

Image Size. Because crossed straight lines connect the object and the image, the angles œ formed by the lines are identical on either side of the pinhole.

Thus if the object and the image were equally distant from the pinhole, the size of the image would be identical to the size of the object.

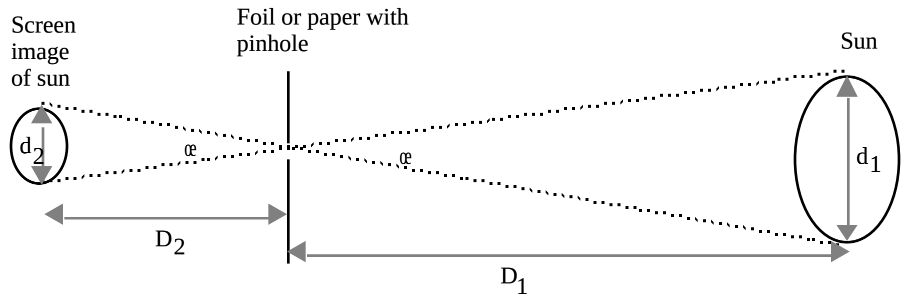


If the image screen moves closer to the pinhole than the object's distance, the image will be smaller than the object. If the object's distance from the pinhole is smaller than the screen's distance, the image gets larger than the object.



Screen located at three different distances from pinhole.

The relative "heights" of the two entities, object and image, are simply given in terms of ratios of the distances and the sizes of the images, as shown below.



$$\frac{d_1}{D_1} = \frac{d_2}{D_2}$$

where d_1 and d_2 are the diameters of the Sun and its image, resp. and D_1 and D_2 are the distances from the pinhole to the Sun and its image, respectively.

This ratio works because the two angles œ are the same, formed by two crossed straight lines. Assume that the distance from the Earth to the Sun, labeled D_1 in the diagram, is approximately 93,000,000 miles, which in SI units is $D_1 = 1.5 \times 10^{11}$ meters.

Measure Your Experimental d_2 And D_2

Materials Needed:

1. A piece of blank white paper mounted on a stiff piece of material
2. A small piece of cardboard
3. A small piece of aluminum foil
4. Metric ruler with millimeter scale

Remember that you will probably get best results if you take your measurements when the Sun is high in the sky. Also, this may be difficult to do by yourself. We advise that you find someone with which to work.

1. Cut a hole in the cardboard that is smaller than your piece of aluminum foil. Cover the hole you have cut with the aluminum foil. Use the tip of a sharp pencil or pin to poke a small hole in the aluminum foil.

2. Hold the pin hole about 100 cm above the screen, which is the blank white paper. *It is important that the cardboard with the pin hole and the screen be perpendicular to the sunlight coming down. Since the Sun is never directly overhead at our latitude (34° º N), you need to tilt the screen until it is perpendicular to the incoming sunlight.* If you find you cannot get a bright enough image at 100 cm, move the hole closer to the screen until you get a satisfactory image. You may have to adjust the size of the "pinhole" to see a reasonably bright image. (At first, just practice, moving the screen closer and farther, tilting it to see various effects of your actions.)

3. Measure the pinhole-screen distance. Call this distance D_1. Measure the diameter of the image, d_1, on the screen.

NAME ________________________________ ID# ___________________________________

PARTNERS ____

1. $D_1 = 1.5 \times 10^{11}$ meters: The distance from Earth to the Sun.

d_1 = diameter of the Sun : The quantity that is to be determined by you.

D_2 = ________________ meters: The measured distance from pinhole to image on screen.

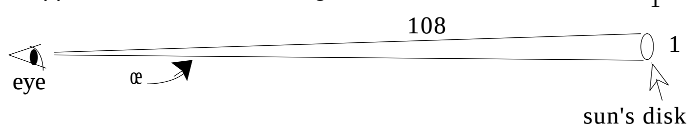
d_2 = ________________ meters: The measured diameter of the circular image on the screen.

a) Determine approximately the diameter of the Sun by calculating the appropriate ratios. Show your work in the space below. (See page 7a-6. Use notation involving powers of ten.)

d_1 = _________________ meters = the diameter of the Sun

b) The diameter of the Earth is $d_E = 1.27756 \times 10^7$ meters. Divide d_E into your value for the diameter of the Sun, in order to compare their relative sizes. About how many diameters of the Earth will fit across the diameter of the Sun? __________________

2. An approximate ratio for the angle that the Sun subtends is $\frac{d_1}{D_1} = \frac{1}{108} =$ 0.00926 radians..



An angle in radians can be converted to degrees, and 0.093 radians approximately equals 0.53 degrees, or about one-half degree. If you recall your measurement of the diameter of the full Moon by using your little finger held at an arm's length , you saw that the Moon's diameter was also about one-half degree. The moon's disk fits rather exactly—and that exactness is pure coincidence—over the sun's disk during a solar eclipse.

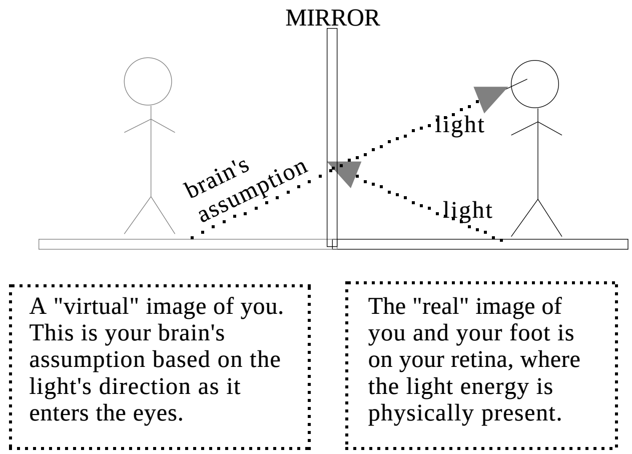
Compare your measured value of $\frac{d_2}{D_2}$ to the accepted number, $\frac{d_1}{D_1} = \frac{1}{108} =$ 0.0093 radians, by subtracting the two values—the measured and the accepted—and then dividing the difference by the accepted value. Obtain the percent difference between your measurement and the accepted value and record it below. Record the "absolute value" of the percent difference, which means simply the magnitude without + or — signs.

%-difference between your measured ratio and 0.0093 = $\frac{0.0093 - \frac{d_2}{D_2}}{0.0093}$ x100% = __________ %

APPENDIX 1: Straight-line Propagation of Light

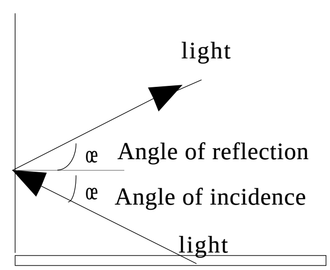
Straight-line propagation of light is so convincing to our visual sense that we often can be tricked into seeing things where they, in fact, are not. For example: Why do you see your image in back of the mirror? Why is the image's distance in back of the mirror identical to your actual distance from the mirror's front?

The answer is in the drawing to the right. Imagine that is you to the right of the mirror gazing intently at your mirrored foot. Some light is illuminating your foot, and one particular path for the light from the tip of your foot to your eyes is shown.



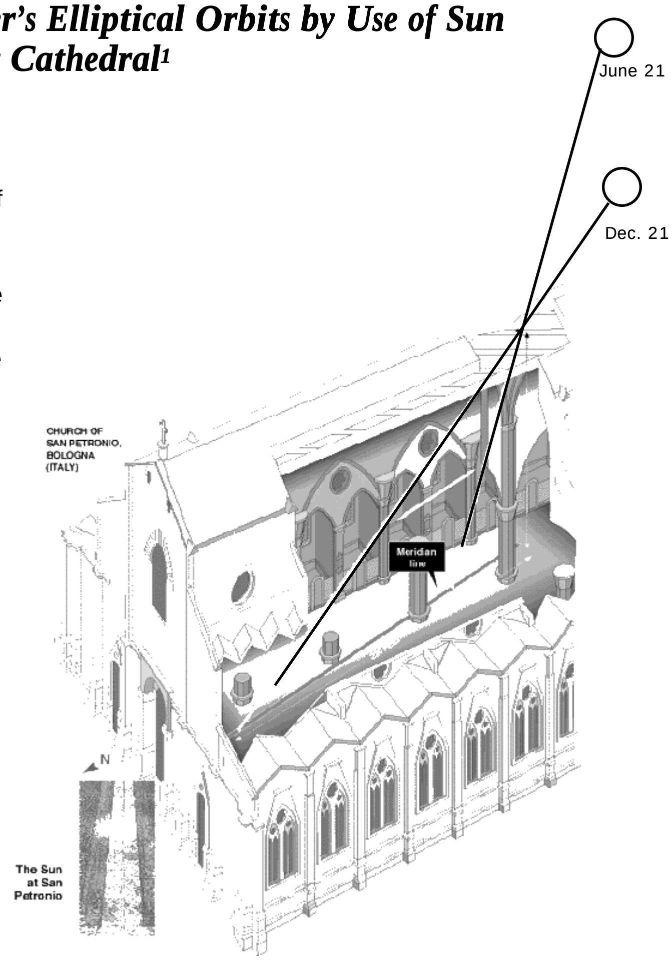
Your eye-brain system makes an assumption, based on your visual experience since you were a child, that light that enters your eye from a particular direction did, in fact, come to you on a straight line along that direction. This "experience" or assumption is apparent when you look into an ordinary plane (flat) mirror and see yourself behind the mirror. Here's how it works:

Your foot reflects light toward the mirror. This is indicated by the "ray" of light drawn from your foot to the mirror. The light moves in a straight line, where it is reflected by a very flat, smooth surface. The light changes direction in a predictable way: the new direction is such that the angle of incidence of the incoming light with the plane mirror is equal to its angle of reflection. Your eye-brain system immediately and unconsciously analyzes and interprets the light as having traveled in a straight line. Therefore, it projects the image of your foot to the place it believes the light "should" have come from. Thus you see an image at a place where nothing of you exists--not even light is at that place. You see your foot in back of the mirror.



APPENDIX 2: Verification of Kepler's Elliptical Orbits by Use of Sun Images on the Floor of a 16th-Century Cathedral[1]

During much of the 16th, 17th, and 18th centuries, Roman Catholic cathedrals were the best solar observatories in the world. That's the conclusion of research by John Heilbron reported in *The Sun in the Church* (Harvard, 1999). The Church's motivation was to pinpoint officially and exactly the date of Easter, which was directly tied to knowing the vernal equinox date. The Julian calendar in use at the time was not very accurate.



In 1655, Giovanni Cassini, an Italian astronomer and a contemporary of Newton, improved the design of an older solar observatory by engineering a hole, approximately 90 feet above the floor, in the roof of the Basilica of San Petronio in Bologna, Italy. It made the grand cathedral into something like a very accurate pin-hole camera. A sun-image sweeps through the cathedral every day, and its position along the floor can be easily observed. A north-south meridian line was established in the tile patterns of the floor. The sun-image would cross such a meridian at exactly noon (this event is actually the definition of noon at a given locale), and was used to set clocks and railroad schedules.

Cassini used the cathedral sun-observatory in 1655 in what is probably the first independent experimental confirmation of Kepler's prediction of elliptical orbits, and it was done with Earth's orbit, which has an eccentricity of 1.7%, which is almost 6 times smaller than Mars' eccentricity at 9.3%. An elliptical orbit means that the Earth-Sun distance is not constant but varies during each year. We know today that we are closer to the Sun in January than in June. Cassini reasoned that the diameter of the disk of the Sun, as it appears in the sun-image on the floor of the cathedral, would be smaller and larger at different times of the year. To make this distinction, Cassini's measurements could have an uncertainty no larger the three-tenths of an inch (0.3 in.) After much trial and error Cassini and scientists associated with the Jesuits succeeded in confirming this inference from Kepler's model of the solar system.

During the next 80 years, astronomers made over 4500 observations of the Sun, greatly increasing the precision of the Earth's orbital relationship with the Sun.

1 The drawing is adapted from one presented in the New York Times by Juan Velasco, 10/19/99.