TAKE-HOME EXP. # 2
A Calculation of the Circumference and Radius of the Earth

On two dates during the year, the geometric relationship of Earth to the Sun produces "equinox", a word literally meaning, "equal night" or equal durations for night and day. The Fall equinox occurs approximately Sept. 21, and the Spring equinox, approximately March 21. On these two days, every year:

March 21 and September 21

a) The time between sunrise and sunset is approximately 12 hours everywhere on Earth.

b) The Sun is directly overhead at noon at the equator. As a consequence, the shadow-terminator line is parallel to a line of longitude, and, therefore, the shadow of Earth bisects each polar region.

This experiment will use the second fact to directly measure your latitude angle. And, as a consequence, you can directly produce the actual circumference and radius of the Earth! The circumference and radius of Earth helps to locate human beings within the context of a universe of galaxies.

A.   A Procedure To Find The Circumference Of The Earth
This procedure was first used by Eratosthenes, who headed the great Museum at Alexandria (or Cairo) in Egypt, around 200 BC

Most of the ancient observers were completely convinced of the roundness of Earth. The evidence was clear. Lunar eclipses were correctly interpreted as showing the circular shadow of the Earth. Also, if one traveled north, the northern constellations got higher in the sky, and the southern ones dropped toward the horizon. A similar effect occurred for southward travel, with the south constellations getting higher in the sky, and even some new constellations, not seen from the northern hemisphere would be visible.

With that background, Eratosthenes also had some additional local information. On the summer solstice date, Eratosthenes knew that the Sun illuminated the bottom of a very deep well at Syene, about 500 miles directly south of the Museum at Alexandria. In order to reach the bottom of such a narrow, deep shaft, the Sun at Aswan must have been nearly directly overhead at noon. If a stick had been oriented vertically at Aswan on that date, it would have cast no shadow at noon on that day.

Formerly, Alexandria

Formerly, Syene

Tropic of Cancer
23.5° N latitude

Note on the map that Syene (Aswan) is just a short distance north of the 23.5° N latitude line, the Tropic of Cancer, which is the farthest north latitude at which the Sun can be directly overhead. That occurs there about June 21, the solstice date.
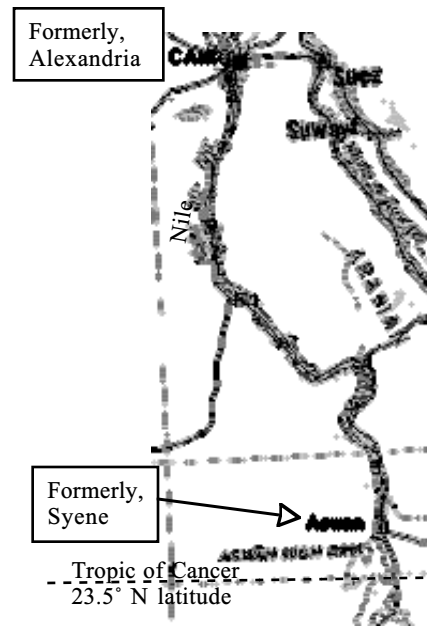
The Use of a Vertical Stick Called a "Gnomon". The use of a vertical stick perpendicular to a flat surface can tell one a great deal about the time of day and the time of year. The ancient Greek word for this device was "gnomon", and it has been directly taken over into English. It should be found in any "collegiate" dictionary.

a) At noon, the shadow of the stick (gnomon) is at its shortest and points directly north in the northern hemisphere. Watching the shadow change size and direction over the course of a day or two provides an accurate compass.

b) If one watches the stick for an entire year, one can note other interesting features of the rhythm of the Sun's appearance. Imagine making a mark representing the shadow length on the plane at noon, say, once a week for a year. If one starts in the Spring, one observes the shadows at noon decreasing in length, getting shorter week by week, until late June. From that time on the noon shadows get longer and longer . . . and longer until late December. Then the shadow again reverses course. One sees two special dates where the shadow reverses course—the Sun seems to "stand still", "solstice" in Latin.

On the solstice date, when Eratosthenes knew the Sun was directly overhead at Syene (Aswan), he did something remarkably simple at Alexandria. *He measured the length of a monument's shadow and the vertical height of the monument.* Then, with his knowledge of the geometry of a circle and a measure of the distance between Alexandria and Syene, Eratosthenes was able to calculate the circumference and radius of the Earth.


Sun's rays

SIDE VIEW

h = height of vertical object

L = length of shadow
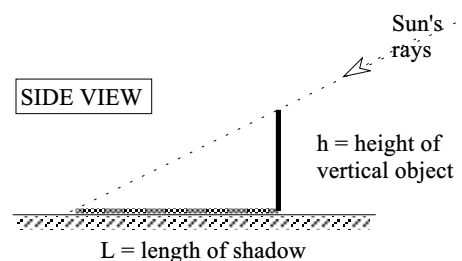
It almost seems like magic that a simple length measurement
locally could turn into a planetary measurement.

B. Here's The Reasoning

When sunlight arrives in the vicinity of
Earth, all the sunlight has the same angle of
arrival. The diagram to the right shows
sunlight arriving directly overhead at Syene
at the summer solstice. At that same moment,
as we look to the north and south of Syene,
all the arriving light is traveling parallel to the
light rays at Syene.
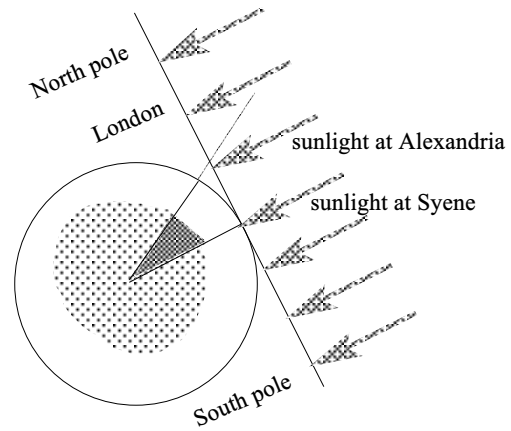
Because the Earth presents a curved surface
to the arriving light, the incoming light makes
different angles with vertical objects at
different locations along the surface.
A vertical object at the Earth's surface
can be defined as one that is parallel
to a radius of Earth at that location.
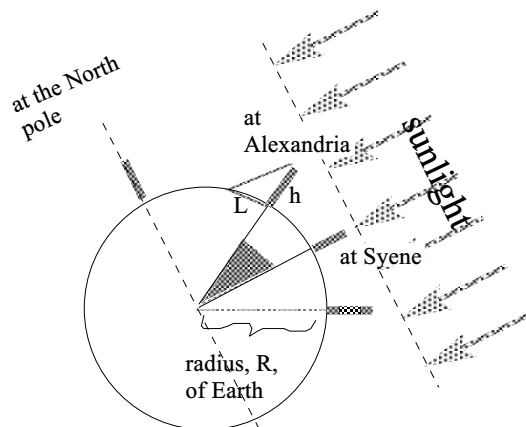
 The curvature of the surface causes
vertical objects, like standing people,
to have *different length shadows at
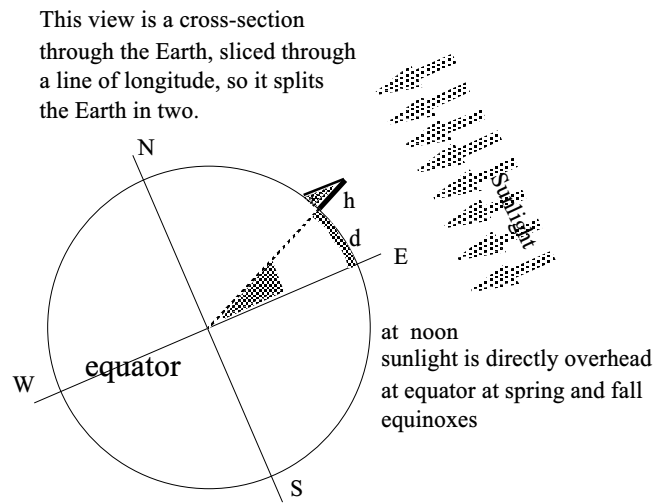different places at precisely the same
moment*.
Eratosthenes measured h and L in
Alexandria *at the moment the bottom
of the well was illuminated in Syene*,
likely to be noon on that date. He also
knew the distance d between the two
cities of Alexandria and Syene.

Vertical objects are located at various
latitudes. The shadow cast by each can
be obtained by using a line parallel to
the incoming rays of the sun. Draw the
line from the top of the object toward
the horizon plane of the object.

Your Analogous Experiment. You will use an equinox date to accomplish the same thing that Eratosthenes did at a solstice date. The "magic" in the equinox date is that we know exactly where the sun is, and exactly when: directly overhead at the equator at noon. The other value that is required will be your "straight-line" distance, labeled "d" in the diagrams, from the equator to your location.

An accurate statement about the equinox date is that sunlight arrives directly perpendicular to the Earth's surface *at the equator at noon* (sun-time, not daylight savings time).

A vertical object of height, h, placed perpendicular to a flat surface can be imagined to be a slight extension of the radius of the Earth. The diagrams to the right examines the geometry created by the placement of the vertical object h and its shadow. The sun's rays are everywhere parallel at noon.

This view is a cross-section through the Earth, sliced through a line of longitude, so it splits the Earth in two.

at noon
sunlight is directly overhead at equator at spring and fall equinoxes

There are two parallel lines, the equator and the incoming sunlight. The radius of the Earth that passes through your location cuts across these two parallel lines. Therefore, the two shaded angles, labeled "A", are precisely equal, as in all such cases of two parallel lines cut by a third straight line.
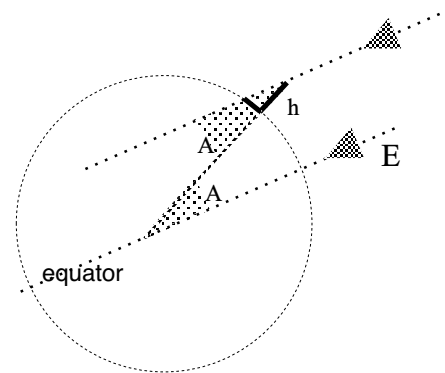
Notice that the angle "A" formed by two Earth radii—one from the center of the Earth to the equator, the other from the center to your location—fulfills the definition of latitude angle. Your location is "A" degrees north of the equator.

The sun's rays are everywhere parallel at noon. Thus, you have two parallel lines, the equator and the incoming sunlight. The radius of the Earth that passes through your location cuts across these two parallel lines. Therefore, the two shaded angles, labeled "A", are precisely equal, as in all such cases of two parallel lines cut by a third straight line.

C. Here's Some Necessary Information About Circles

a) As for any circle, once around the full circle of the cross-section of the Earth covers an angular distance of *360°*.
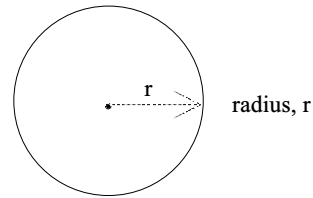
b) The circumference of any circle can be computed from the radius :

$$\text{circumference} = 2\,\pi\,r \, ,$$

where $\pi = 3.14$, and r = radius of Earth.

radius, r

Imagine a clock with only one hand pointing to the numeral "3" position. If you move that hand once around, returning it to the "3" position, you will have moved the hand through an angle of 360°.

The circumference is the length a piece of string would have by laying it carefully along the outline of a circle until you have covered the outline exactly and completely. The circumference of a circle is simply a length in meters.

D. Measuring Your Latitude Angle

The major requirement is to construct a vertical object of known height, h, perpendicular to a flat, horizontal surface onto which the shadow of the object is cast. A sloping reference level will not provide a shadow of "true " length.

Sun

Angle A

h = height of vertical object

L = length of shadow
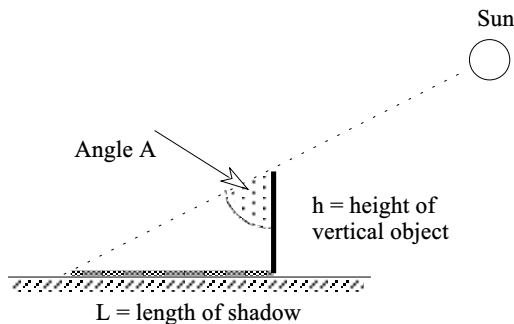
A meter stick or a 12-inch ruler will work well, but any straight object that is vertical will work. (I've used fence posts, books, and even a pen, as my vertical object. Let's label whatever is used as a "stick.")

This measurement needs to be done on the equinox date at noon, sun time, not daylight savings time. If daylight savings time is in effect, do the measurement at 1 PM. A time range of "noon ± 10 minutes" will make little difference in the outcome.

Doing it at noon, sun time, a few days on either side of the equinox date will also make little difference. If the weather doesn't cooperate, do it as soon as it does.

The angle labeled "A" is "the angle of the incoming sunlight" measured from the vertical.

THE REQUIREMENTS: The height h must be at 90° to the surface, and the surface on which the shadow is cast must be horizontal. It cannot be sloped up or down at all. The height h and the length, L, of the shadow needs to be measured.

Angle A is the same as latitude angle.

sunlight

A

h, height of vertical object

90°

L = length of

HELPFUL HINTS: You can measure the vertical alignment of h be comparing it to a "plumb line", which is simply a string tied to some weight (your keys, for example) and the weight is allowed to hang freely. The string is then vertical by definition.

Also very important: The shadow's length must be carefully considered. Note in the diagram to the right that the shadow's length L is measured from the leading edge of the object, not from its center or back edge.

Measure "L" from front edge of extended object.

h

L
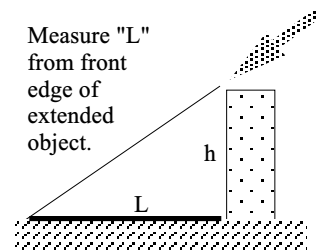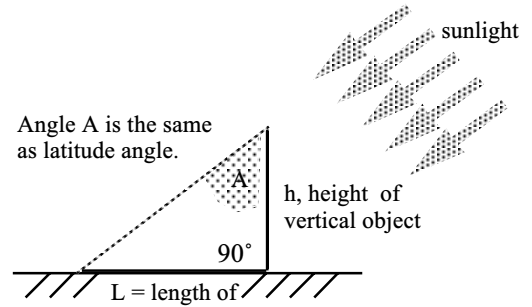
NAME (print)_____ ID# _____

PARTNERS (if any) _____

DATA SHEET

Measuring Your Latitude Angle **Time and date of measurement: _____**

Measure the shadow length L and the vertical height h, including an estimated uncertainty:

h = _____±_____ cm   Shadow L = _____±_____ cm

Divide L by h. $\dfrac{L}{h}$ = _____

Then compute angle A by doing the following using your calculator.

$$A = \text{angle} = \tan^{-1}\left(\frac{L}{h}\right) = \underline{\hspace{3cm}} \text{ degrees.}$$

HERE'S HOW: With the result of $\dfrac{L}{h}$ in your calculator window, use the inverse tangent operation ($\tan^{-1}$) on your calculator. To do so usually requires hitting the "2nd function" button on the calculator to access this function, which is located above the "tangent" button.

[Angle "A" should be very close to the northern latitude angle at which you made the measurement. If it's not, either something is wrong with your measurement, or your calculator is putting out some other angular measure, like radians or grads, both of which are directly proportional to degrees. There is a button somewhere on your calculator which will allow you to switch modes between degrees, radians, and grads. Switch to degrees and try again.]

C. *Estimating A Relatively Accurate Circumference And Radius Of Earth*

Please assume that the distance "d" from the equator to your location somewhere near Los Angeles is 2350 ± 50 miles. If you are more than 100 miles north or south of Los Angeles, please adjust the distance value.
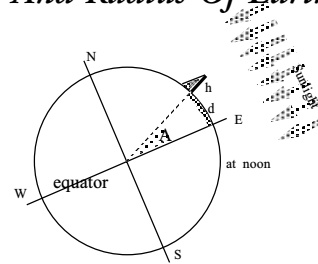
Next, from the information you have in this experiment, please obtain the circumference and the radius of the Earth. Show your reasoning and work. Explain how you obtained the values

.

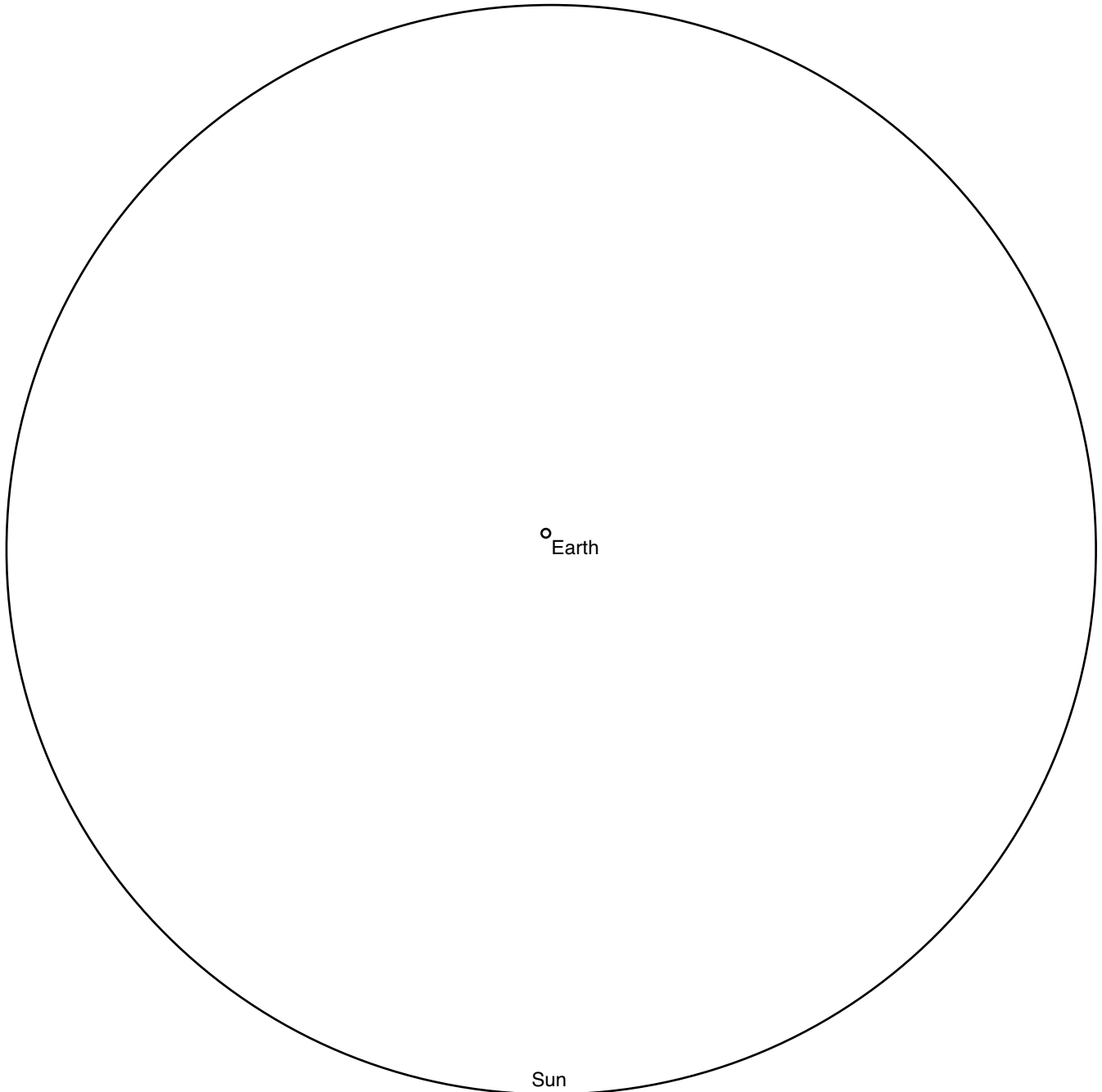Circumference of the Earth = _____±_____ miles

Radius of the Earth = _____±_____ miles

% difference between measured value of the radius r and the accepted value   (See Endnote 1)

$$\frac{\left| r_{exp} - r_{accepted} \right|}{r_{accepted}} \times 100\% = \underline{\hspace{3cm}} \%$$

Appendix 1. The Reason The Light Arrives In Parallel All Over The Earth

Some people think of the Sun as occupying a relatively small portion of the sky, more akin to a point source of light than to an extended "wall of light" covering the entire sky. The apparent disk of the Sun is no bigger than the apparent disk of the Moon, each occupying about 0.5° in the sky. This perception bears some examination. The geometric reality of sunlight is something quite different. In geometric fact, the actual diameter of the Sun is about 100 times the diameter of the Earth. This 100-to-1 geometry governs the physical path of the light. Even though the Earth is a distance of about 200 sun-diameters away from the Sun, the light from the Sun arriving at Earth is light from a very large extended source. The Earth intercepts only a tiny portion of the light emitted. Below are two circles with a ratio of diameters 100 to 1.

[1]The "accepted" average value for Earth's circumference is C = 24,906 miles, and for the radius r = 3964 miles.
   A careful measurement on the appropriate dates should get one to within about 5% of these values.