# Introductory Lectures: The Nature and History of Cognitive Science

# Chapter 1: Scientific Treatments of Physical Domains

By Dr. Charles Wallis Last Revision 1/25/2014

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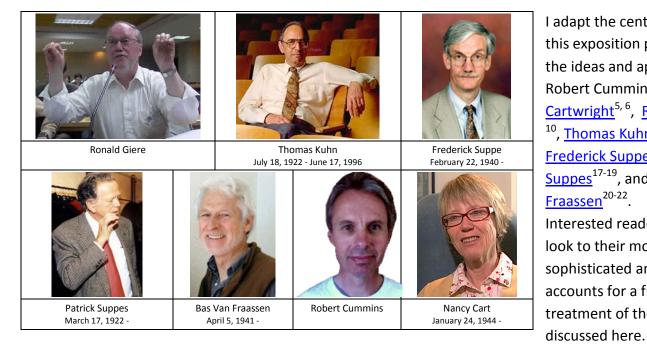
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# 1.1 Questions, Answers, and Frameworks: Developing Scientific Treatments of Domains

All human beings spend some time wondering about the nature of the world, their own nature, and how the two fit together. Asking such questions marks the beginning of rational inquiry. However, questioning in itself is not inquiry. Inquiry requires adopting a framework—answering certain questions—so that one has a basis for answering others. For example, science itself represents an answer to questions about the sorts of experiences that can count as evidence, how one ought to categorize phenomena, the sorts of dynamical

relationships constitutive of an adequate treatment of phenomena, the manner in which one ought to test dynamical hypotheses, etc.. Indeed, scientific knowledge differs from religion or commonsense, for example, in that science insists upon adopting a framework within which researchers address questions regarding phenomena through highly controlled, repeatable experimentation. Cognitive science seeks answers to many questions at many levels of description, and like any other science, it seeks answers to these questions by working within a framework that adopts common answers to certain central questions.

Before turning to cognitive science specifically, this chapter outlines a general picture of the products and processes of science. This general framework provides students with a perspective on the nature of science (and specifically of cognitive science) allowing them to better understand the process of scientific development as well as the specific details of current theories. The general framework for understanding scientific processes and products that I introduce in this chapter likewise structures the presentation of material throughout the text and lectures. The account given here no doubt seems somewhat superficial to the more sophisticated, and many philosophers might disagree with various elements. Nevertheless, I offer it as a useful schema for understanding science and the scientific process.



I adapt the central ideas in this exposition primarily from the ideas and approaches in Robert Cummins<sup>1-4</sup>, Nancy Cartwright<sup>5, 6</sup>, Ronald Giere<sup>7-</sup> 10, Thomas Kuhn<sup>11-13</sup>, Frederick Suppe 14-16, Patrick Suppes<sup>17-19</sup>, and Bas Van Fraassen<sup>20-22</sup>. Interested readers should look to their more sophisticated and thoughtful accounts for a fuller treatment of the issues

## 1.1.a Ontological Frameworks

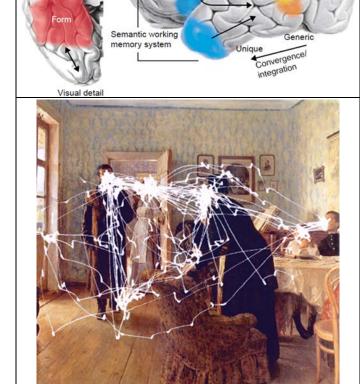
Cooperative investigation and theorizing requires formulating and agreeing upon basic ideological and methodological constraints within which researchers conduct inquiry. Thus, a prerequisite for any theorizing involves researchers formulating (and achieving a rough consensus within their community) a general framework for understanding the nature of a domain and its phenomena. Perhaps the most fundamental presupposition of any inquiry, therefore, concerns what I call an ontological framework. Ontological frameworks act to constrain and focus investigation in large part by providing a set of fundamental categories, generalized relationships, and methodological practices within which one can formulate meaningful questions and propose theoretic answers to those questions. For instance, most people do not think that logically impossible situations can serve as counterexamples to a theory. So, when told that circles consist of sets of points equidistant from a center point on a Euclidean plane, it strikes people as irrelevant to object, "But what if the circle is a square?" Thus, philosophers generally agree upon the constraint that counterexamples to

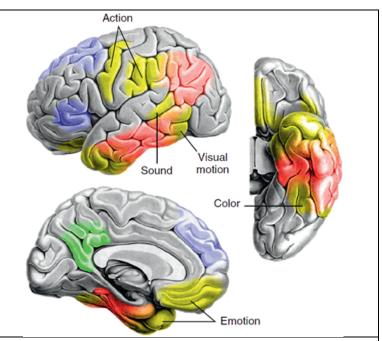
theories must pass the minimum standard of logical possibility. Similarly, scientists accept that statistically significant finding must meet the minimum standard of .95 probability--meaning the probability of the experimental result occurring by chance alone must be less than .05.

## 1.1.b Fundamental Categories, Dynamical Interactions, and Attributions

An important part of understanding science and scientific practice involves recognizing that scientific practice **does not** represent an alternative methodology for thinking about the world. Rather, scientific practices build upon features and methods found in cognition generally. For example, humans naturally differentiate objects in the world into the categories of living from non-living entities; they likewise differentiate and understand phenomena using these terms as well. In fact, people can develop a deficit for naming non-living things, while remaining relatively unimpaired in naming living things. Indeed, the very structure of the brain's semantic memory provides the basis for some of this asymmetry.<sup>23-26</sup> Likewise, humans appear to manifest an

motion artifact





(**Top left**) A diagram from Martin and Chao depicting the approximate brain regions associated with semantic memory storage and retrevial for object form, motion and object-use-associated motor patterns. (**Above**) Diagram from Binder and Desai depicting one model of semantic processing. Yellow indicates modality-specific systems providing input into higher-level convergence areas (in red) responsible for abstract representations events and entities. Blue areas depict brain areas associated with the goal-directed activation of stored information. Finally, green areas indicate brain region hypothesized to to mediate semantic storage areas and the hippocampal memory system, possibly facilitating the creation of episodic memories. (**Bottom Left**)The visual saccade path during "free" or undirected scanning imposed upon the painting. Sasha Archibald, after Alfred Yarbus, after Ilya Repin, They Did Not Expect Him (aka An Unexpected Visitor, 1884) Notice that the faces receive the greatest number of saccades during the viewing period. From: Observations on Film Art.

innate disposition to categorize objects and phenomena into mental and non-mental entities and/or phenomena. This disposition to think about the world in terms of mental and non-mental entities occurs in even in early, automatic, unconscious perceptual processing, driving behavior and manifesting itself early in development. For instance, faces strongly attract visual attention (saccades). The human visual system's preference for faces occurs at the very earliest stages of scene perception when the brain selects objects to which to attend. This preference for faces manifests itself by 3 months in human infants—suggesting an

innate disposition to find faces visually salient.<sup>43-46</sup> Likewise, humans automatically and unconsciously process information regarding the emotional states and motor intentions of other people during vision. Indeed, "mind blindness" is one of the most significant pathologies associated with autistic spectrum disorder.<sup>27-39</sup> Humans, moreover, monitor and interact with other people using a vast array of automatic and unconscious processes.<sup>42, 47-49</sup> In short, the human disposition to categorize the world into mental and non-mental results—at least partially--from a variety of innate, automatic, and unconscious cognitive processes.

As the lectures and text will emphasize repeatedly, philosophical and scientific theories of the mind throughout history attempt to understand and to either affirm or deny the real-world basis of this innate tendency of human categorization. Do the categories of mental and non-mental cut the world at a joint? That is, does the distinction marked by the mental versus non-mental categorization correspond to a real and important distinction in the world? Must any adequate theory of the number and nature of the universe's basic kinds recognize the existence of mental and non-mental objects, properties, etc.? Within the framework of specific answers to such questions philosophers and scientists strive to systematically formulate, observe, and theorize about mental phenomena and entities as well as to characterize the place of mental phenomena and entities in relationship to physical phenomena and entities. In other words, if these categories mark a fundamental and real distinction between kinds, what relationship do these kinds have to one another?

All of the thinkers in this text and in this course explicitly or implicitly adopt and theorize within this most basic of constraints upon inquiry--an ontological framework. Specifically, fundamental categories tend to specify the kinds of things and the kinds of changes considered legitimate (real). Part of this chapter outlines how theorists utilize the categories, types, and interrelationships of an ontological framework to specify a domain of inquiry and to formulate theories and test theories. Finally, I discuss how ontological frameworks transition from a position of high salience early in inquiry to the status of a rather amorphous and neglected theoretic purlieu within more advanced inquiry.

As a scientific treatment of a domain develops, the period during which ontological frameworks play the most significant role, and seem most conspicuous, is during the early stages of inquiry or during significant upheavals in inquiry (like periods sometimes characterized as scientific revolutions). As the chapters on the development of philosophical and psychological theories of cognition make manifest--philosophical and scientific theorists throughout history have devoted a great deal of their energy towards understanding how best to categorize mental phenomena and towards understanding the place of mental phenomena in relationship to physical phenomena. Do the mental and the physical constitute distinct kinds of things—fundamental categories--or do they fall into a single kind? Do the processes and changes operant in the dynamic temporal evolution of mental phenomena differ fundamentally from the processes and changes driving the temporal evolution of physical phenomena? One can find similar questions and debates regarding the proper ontological framework for understanding the nature of living vs non-living things and the relationship between organic and inorganic processes in the development of biology. We'll return to this issue in the chapters on the development of philosophical and psychological theories of mind.

In more developed sciences ontological frameworks recede in prominence as researchers shift their focus from broad framing issues towards the development of specific theories. Nevertheless, ontological frameworks continue to operate as constraints upon theory formulation--by articulating a hypothesis regarding number and the nature of the fundamental categories for some domain or domains. However,

during periods of dramatic theoretical shifts ontological frameworks often return to prominence as researchers reexamine even the most basic presuppositions of their science. For example, the rise of quantum mechanics represents a change in physical theories that challenged many of the tacit, but widely and deeply held tenets of the ontological framework that had guided physics since the renaissance. Einstein's famous proclamation in his 1926 letter to Max Born concerns ontological frameworks—not specific theoretic details:<sup>50</sup>

Quantum mechanics is certainly imposing. But an inner voice tells me that it is not yet the real thing. The theory says a lot, but does not really bring us any closer to the secret of the 'old one'. I, at any rate, am convinced that He is not playing at dice. (p.90)

For Einstein the inherently probabilistic nature of much of quantum mechanics violated an important constraint--physical theories must be mechanistic and deterministic. Indeed, the nature of the disagreement between Einstein and Born comes most clearly into focus in Einstein's 1950 letter, where he states that:<sup>50</sup>

I see from the last paragraph of your letter that you, too, take the quantum theoretical description as incomplete (referring to an ensemble). But you are after all convinced that no (complete) laws exist for a complete description, according to the positivistic maxim *esse est percipi*. Well, this is a programmatic attitude, not knowledge. This is where our attitudes really differ. For the time being, I am alone in my views as Leibniz was with respect to the absolute space of Newton's theory. There now, I've paraded my old hobby-horse once again. But it is your own fault, because you provoked me. (pp. 188-9)

Fundamental categories consist of the set of categories considered essential and ineliminable to any adequate account of the phenomena in some domain. These categories further constrain the sorts of attributions and dynamical interactions theorists can utilize. Prior to general relativity, for instance, physicists consider space and time to be distinct elements of the universe. After general relativity, space and time become a single element space/time. Similarly, the ontological framework of modern physics includes the category of force. Thus, modern physicists claim that adequate theories of physical phenomena must include forces.

The category of forces illustrates some important aspects of an ontological framework. First, some of the elements of an ontological framework prove less central than, even dependent upon, other elements. Indeed, modern physics recognizes two general categories of forces. On the one hand, physicists appeal to "contact forces." Contact forces transfer energy by direct mechanical contact. For example, friction is such a force. On the other hand, physicists also posit the category of "fundamental forces." Fundamental forces (sometimes called field forces or interactive forces) constitute the current hypothesis as to the number and nature of essential and ineliminable forces in modern physics. Thus, contact forces prove dependent upon fundamental forces in that all contact forces ultimately result from fundamental forces acting on objects. For example, friction at the pivot of a pendulum results from the surfaces dragging against one another during the swing of the pendulum (see Diagram A below). The swing itself as well as the contact pressure that results in the drag (the frictional contact force) comes from gravitation (a fundamental force).

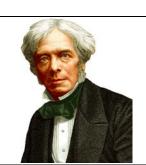
Currently physicists recognize four fundamental forces; gravitation, <sup>51</sup> electromagnetism, <sup>52</sup> strong nuclear force, <sup>53</sup> and weak nuclear force. <sup>54</sup> Fundamental forces illustrate a second important point regarding ontological frameworks; the elements and properties of an ontological framework can change as inquiry progresses. In the history of physics, the number and nature of fundamental forces can and has increased and

decreased as theories change over time. Indeed, prior to <u>James Clarke Maxwell's</u><sup>55</sup> publication of "On Physical Lines of Force," in 1861 and <u>Treatise on Electricity and Magnetism</u> in 1873.<sup>56, 57</sup> Physicists treat electric force and magnetic force as separate fundamental forces. Today, however, physicists posit a single force in relation to both electric and magnetic phenomena--the electromagnetic force.

What brought about the change? Physics begins by treating electric and magnetic phenomena as unrelated. As a consequence physicists propose electric and magnetic forces in explaining the respective phenomena. However, as time goes by physicists begin to notice relationships between electric and magnetic forces. Maxwell's book represents a synthesis of work that begins around 1820 with the Danish chemist and physicist Hans Christian Ørsted. For the property of the physicist of the property of the physicist in the physicist of the physicist in the physicist of th



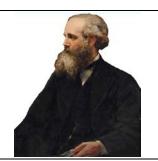
Hans Christian Øersted (1777–1851) From: Wikipedia



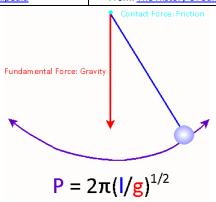
Michael Faraday (1791–1867) From: The History Of Surgery



Sir Humphry Davy (1778-1829) From: Wikipedia



James Clerk Maxwell (1777–1851) From: Your Paintings



**Diagram A** illustrating how both contact and fundamental forces operate in pendulums. Since all contact forces result from fundamental forces acting on objects, the pendulum's swing as well as the contact pressure that results in the drag at the arm pivot result from the fundamental force of gravitation (in red). The contact force, friction (in light blue), operates at the pendulum arm pivot resulting from the surface drag during the swing of the pendulum.

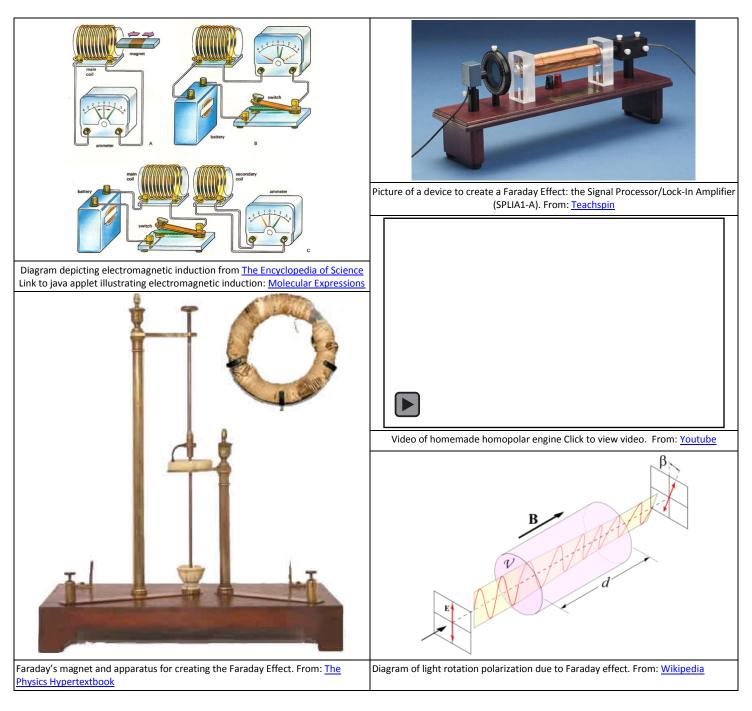


Diagram B Picture of "A small (~6mm) piece of pyrolytic graphite levitating over a permanent neodymium magnet array (5mm cubes on a piece of steel). Note that the poles of the magnets are aligned vertically and alternate (two with north facing up, and two with south facing up, diagonally)." This is an example of diamagnetism.

Description and picture from: Wikipedia

The next significant contribution to the unification of electric and magnetic forces comes from the work of Michael Faraday, <sup>60</sup> an English chemist and physicist. Faraday attends a lecture given by the English chemist Humphry Davy. <sup>61</sup> Faraday is so impressed with Davy, Faraday seeks employment in Davy's lab. Faraday submits a letter together with a 300 page book based upon notes from Davy's lectures. Davy hires him, first as a secretary, and later as an assistant. Faraday designs experiments that result in the <a href="https://doi.org/10.1006/journal.org/">https://doi.org/10.1006/journal.org/</a> (an electric motor with a fixed magnetic polarity), reveal <a href="electromagnetic induction">electromagnetic induction</a> (the flow of an electric current through a conducive medium [like a wire] by changing the electric field), <a href="mailto:diamagnetism">diamagnetism</a> (the property

of some materials to create an opposing magnetic field when one applies a magnetic field to that material; see diagram B on page 2 above), and that show that magnetic forces can affect light (the Faraday Effect)<sup>65</sup>. Faraday also argues that electric phenomena result from a single kind of electricity and that electromagnetic forces extend beyond the physical conductor. His contemporaries reject much of his work, in part because he lacks the mathematical knowledge to express his theories mathematically. <sup>60, 66, 67</sup>

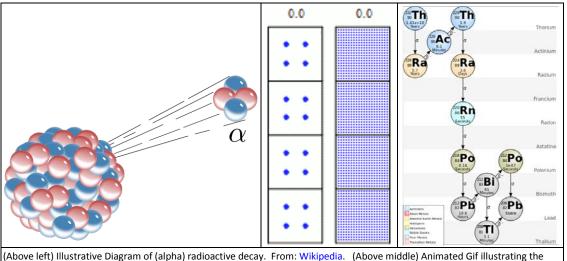


Finally, Maxwell publishes his <u>Treatise on Electricity and Magnetism</u> that includes four laws.<sup>57</sup> These laws together form the basis of classical electrodynamical theory. One of the laws expresses Faraday's results on electromagnetic induction. Maxwell's work unifies electric, magnetic, and light phenomena, showing these diverse phenomena result from the same entity, electromagnetic fields that travel through space as waves moving at the speed of light.

In summary, ontological frameworks provide structure and constraints upon inquiry by forwarding a hypothesis regarding number and nature of the fundamental categories for some domain. Fundamental categories serve in an ontological framework as the essential and ineliminable elements in any adequate account of the phenomena in some domain. They also constrain the sorts of attributions and dynamical interactions theorists can utilize to explain phenomena. For instance, fundamental forces like gravitation can interact with objects directly and indirectly through contact forces like friction. Thus, the category of fundamental forces also specifies the sorts of allowable dynamical interactions between forces and between forces and objects. Fundamental forces like gravity constrain attributions as well. Physicists attribute gravitational attraction to all objects having mass and measure this attraction in units like the kilopond (kp), which is equal to the force exerted by one kilogram in 1g (9.80665 m/s² the standardized average of Earth's gravity). As the example of forces in physics illustrates, some of the elements of an ontological framework prove less central, even dependent upon, other elements. Likewise, the nature and number of elements of an ontological framework can change as inquiry progresses.

# 1.1.c Selecting Scientific Domains

In addition to selecting an ontological framework, theorists must also select a set of phenomena that they suppose constitute a scientific domain. How do scientists determine what counts as a domain? The answer I propose to this question no doubts runs contrary to many people's views. I suggest that the process of defining a scientific domain begins when researchers identify potential domains of interest--that is, domains in which the phenomena appear to exhibit both a unity and regularity of interest to theorists. The unities and regularities needed to highlight a set of phenomena as a potential domain for scientific treatment can manifest themselves in the dynamical interactions (how elements of the phenomena interact with one another) and/or the development of domain elements over time (e.x. changes in the properties exhibited by



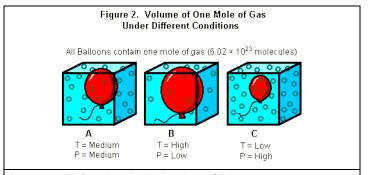
(Above left) Illustrative Diagram of (alpha) radioactive decay. From: Wikipedia. (Above middle) Animated Gif illustrating the rate of radioactive decay of identical atoms. Left boxes four atoms. Right boxes 400 atoms. From: Wikipedia. (Above right) Diagram illustrating the order and types of atomic transmutations in the decay of Uranium. From: Wikipedia.

objects, properties, events, or relations within the domain). While unified and regular dynamic interactions give rise to dynamical theoretical models, attributional theoretical models capture unity or regularity manifested as attributes of

objects, events, or relations within the domain.

A simple example of a unity or regularity manifested in attributes of an object is the rate of radioactive decay for various elements. Scientists identify several "classical" types of <u>radioactive decay</u>. <sup>68</sup> In the most common type of decay a part of the atom's nucleus breaks away resulting in a loss of energy and the transmutation of the atom from one kind of element to another kind of element. <sup>68</sup> For instance, uranium decays through a series of steps into lead. Scientists quantify radioactive decay using the measure of a <u>half-life</u>. <sup>69, 70</sup> The half-

life of a given element consists of the time that it takes for have of the atoms of a given element to decay. The half-life of an element is a regularity that the element manifests, and about which scientists create attributional models.



Theorists usually formulate the ideal gas law as follows: PV=nRT. P=pressure, V=volume, n=the number of moles (the amount of a gas), <math>R=the ideal gas constant, and T=the temperature. For predicating the volume of a gas, as above, one can modify the equation as follows: V=nRT/P. Diagram from: EPA

The <u>ideal gas law</u><sup>71</sup> represents a dynamical theoretic model of the relationship between the pressure, volume, and temperature of a gas. The ideal gas law proves useful for determining how changes in pressure, volume, and temperature affect a gas under a variety of circumstances. In the diagram (left), the law can help to predict how changes in temperature affect volumes of an elastic container like a balloon.

While the nature on the phenomena in a potential domain strongly determine if and how scientists

approach the process of domain building, the interests of scientists, the needs of science and industry, the available experimental techniques, the available modeling tools, and the available data regarding the phenomena and its relationship to other phenomena all make important contributions in formulating and refining a scientific domain. As we will see, Galileo had different interests, experimental techniques, modeling tools and data when he formulated the <u>ideal pendulum law</u>. He viewed the law as an instance of uniform acceleration in a gravitational field. Galileo's experiments seem crude now, his data proves incomplete, and he lacks the sophisticated mathematical techniques available to contemporary physicists. Researchers like Christiaan Huygens, Ibhn Harrison, Henry Kater, and Léon Foucault who follow Galileo introduce better experimental techniques and modeling tools. The applications of pendulums in science and industry drive the interests of these later researchers. The changes that these researchers introduce lead to a slow reformulation of the pendulum law and its domain into what contemporary physicists now call oscillating systems theory. How do scientists coordinate all of this? In the next section we look at the central organizational concept for scientific domains in the work of Thomas Kuhn—the paradigm.

# 1.2 Paradigms and Paradigmatic Cases

The concept of a paradigm occupies a central role in the exposition in this chapter and lectures. Specifically, the notion of a paradigm structures my discussion of how scientists delimit scientific domains and develop theoretical treatments of the phenomena therein. My use of paradigms differs in some ways from Kuhn's use of the term in his writings. Moreover, many theorists object to Kuhn's characterization of paradigms and his use of them in his account of scientific theorizing. Since the term paradigm brings a rather large amount of baggage, I'll take a moment to clarify Kuhn's original notion and use of paradigm as well as to contrast Kuhn's conceptualization and use with my own.

#### 1.2.a Kuhn's Use of the Concept of Paradigms

Thomas Kuhn famously appeals to the notion of a paradigm as the central concept in his account of science.<sup>11</sup> Kuhn uses the concept of a paradigm in two general senses: On the one hand, Kuhn thinks of paradigms as exemplars--specific cases of scientific research and theorizing having two important properties. Paradigms prove both "sufficiently unprecedented" and highly successful in treating some class of phenomena. At the same time paradigms exhibit unresolved problems and/or potential, providing the basis for further work to

remedy difficulties and extend the central insights. On the other hand, Kuhn thinks of paradigms as frameworks abstracted from those specific cases. These frameworks provide the basis for "normal science." Kuhn uses the term, "Normal science," as the moniker for those periods in scientific development during which researchers seek to extend the categories, theoretic models, operationalizations, experimental techniques, etc. of a paradigm to new phenomena and to problematic cases. Kuhn characterizes paradigms in both senses as follows:<sup>11</sup>

Aristotle's *Physica*, Ptolemy's *Almagest*, Newton's *Principia* and *Opticks*, Franklin's *Electricity*, Lavoisier's *Chemistry*, and Lyell's *Geology-these* and many other works served for a time implicitly to define the legitimate problems and methods of a research field for succeeding generations of practitioners. They were able to do so because they shared two essential characteristics. Their achievement was sufficiently unprecedented to attract an enduring group of adherents away from competing modes of scientific activity. Simultaneously, it was sufficiently open-ended to leave all sorts of problems for the redefined group of practitioners to resolve.

Achievements that share these two characteristics I shall henceforth refer to as 'paradigms' a term that relates closely to 'normal science.' By choosing it, I mean to suggest that some accepted examples of actual scientific practice--examples which include law, theory, application, and instrumentation together provide models from which spring particular coherent traditions of scientific research. (p.10)

Kuhn elsewhere describes paradigms and their role in the development of sciences—contrasting the role of paradigms in emerging sciences with their role of more mature sciences as follows:<sup>11</sup>

...somehow, the practice of astronomy, physics, chemistry, or biology normally fails to evoke the controversies over fundamentals that today often seem endemic among, say, psychologists or sociologists. Attempting to discover the source of that difference led me to recognize the role in scientific research of what I have since called "paradigms." These I take to be universally recognized scientific achievements that for a time provide model problems and solutions to a community of practitioners. (p.viii)

In order to minimize terminological confusion, I adopt the convention of using the term paradigmatic cases to pick out paradigms in this sense of specific cases. Such specific (paradigmatic) cases include, at a minimum, three elements: categorizations of the targeted phenomena into kinds of objects, properties, events or relations; operationalizations—methods, techniques, operations, and/or instruments used to tie categorizations to the phenomena through qualitative or quantitative measurements that prove systematic, inter-subjective, and reliable; and theoretical models—structured combinations of categorizations that allow for prediction, retrodiction, manipulation, and explanation of target phenomena. On the other hand, Kuhn also refers to paradigms as the shared "rules and standards for scientific practice" (p.11) abstracted from paradigmatic cases and adopted by researchers as definitive of the scientific domain.<sup>11</sup> I use the term paradigm to designate the general framework that researchers develop around paradigmatic cases for defining a scientific domain and treating the phenomena therein.

Both notions of a paradigm operate in Kuhn's exposition. However, Kuhn resists any analysis of paradigmatic cases into a set of constitutive concepts, theories, instruments, methodologies, values, and similar explicit and/or tacit commitments. Kuhn acknowledges that elements of paradigmatic cases like concepts, theories, and instruments unite and define researchers within the paradigm as well as delimiting the domain itself. Nevertheless, Kuhn holds that the primary unit of analysis remains the individual paradigmatic cases—not the

elements of those cases. Kuhn chooses paradigmatic cases as the basic unit through which one ought to analyze science because of what he views as the primacy and unity one finds in these cases. Kuhn notes that all or nearly all theorists in a domain recognize the salient paradigmatic cases and they likewise structure their theoretical understanding of the domain around these cases. However, Kuhn suggests that a deeper analysis of paradigmatic cases seeking a common set of tacit constitutive elements inevitably proves problematic due to the tacit nature of these elements together with the variation in understanding and values among individual scientists.

For Kuhn, the tacit, ephemeral, and equivocal nature of doctrine as dictated through paradigmatic cases renders such analyses difficult at best and unhelpful at worst. To understand Kuhn's concerns, one can usefully think of paradigmatic cases on the analogy of perception. All scientists "see" the same paradigmatic cases—just as everyone looking at a face sees the same face. However, people may differ in how they interpret what they see when looking at the face. Likewise, they may be unaware of aspects of their reaction or disagree as to why they react as they do. For instance, people may agree that a face is attractive without agreeing as to why the face is attractive. Some people might cite hair color as a reason for attractiveness, others might emphasize eye color, still others might remark upon complexion, etc.. Moreover, what people cite as making one face seem attractive might differ from what they notice in finding another face attractive. Thus, one might well disparage attempts to analyze attractiveness into some definitive set of elements and their relationships. Kuhn believes that the same lack of unanimity holds true for scientists with regard to paradigmatic cases. Theorists may agree on the importance of certain paradigmatic cases, but differ as to why these cases are important or what features of the cases make them important. Thus, Kuhn doubts the value of generalized analyses of paradigmatic cases, particularly in that he views such analyses as attempts to generate a singular, detailed prescriptive methodology for scientific practice.

#### 1.3 Paradigms as Frameworks

In this sense, the exposition of this chapter and lectures proves decidedly un-Kuhnian. Indeed, the current chapter seeks to highlight several elements of a paradigm in Kuhn's second, framework sense. Specifically, this chapter and lecture outline elements of a paradigm that must come together in order for a paradigmatic case (in Kuhn's first sense of an exemplar) to emerge. These elements likewise structure the presentation of paradigmatic cases in textbooks and the general manner in which theorists seek for further elaborate and refine their treatment of a scientific domain. I defend my deviation from Kuhn by noting three general facts. First, Kuhn himself engages in analyses of paradigms. For instance, Kuhn analyzes and compares paradigmatic cases on the basis of the categorizations they employ. Indeed, Kuhn enjoys a certain infamy in some quarters for arguing that successive paradigms often categorize the same phenomena in ways that prove incommensurable with one another. Second, the analysis of paradigmatic cases does not equate to the formulation of a singular, detailed prescriptive methodology for scientific practice. This second point finds excellent illustration in the third fact: Kuhn's frequent analogy to perception, specifically his comparison of paradigmatic cases to Gestalts—perceptual wholes or unified entities not analyzable to their constitutive components--proves antithetical to his view.

To wit, people's perceptual judgments of facial attractiveness look like Gestalts—judgments having no principled decomposition into constitutive elements and their interrelationships. However, one can find research into perceptual judgments of facial attractiveness beginning in the 1970s. This research provides a deep and useful analysis of the elements that drive people's perception of attractiveness, without producing a

singular, detailed prescription for attractive faces. Indeed, despite the apparent disunity in what people say makes a face attractive, several physical facial features provide strong constraints upon the perception of attractiveness. For instance, facial characteristics like pupil dilation, averageness (mean values) of features, symmetry of features, skin color, skin texture, as well as gender-specific dimorphisms (two forms distinct in structure within a single species) heavily influence judgments of attractiveness. Additionally, situational and idiosyncratic factors like familiarity, imprinting during development, hormone levels, fertility cycles in women, major histocompatibility complex dissimilarities (the degree of dissimilarities in immune responses that can prove compatible in an individual resulting from reproduction), peer evaluations, self-perceptions (of attractiveness and personality characteristics), social status, and social learning all modulate impact of physical facial features. This diverse and complex set of factors defies any singular, detailed prescriptive methodology for determining inter-subjective attractiveness ratings. Nevertheless, the depth of insight this research provides into attractiveness judgments together with its unprecedented nature qualifies it, ironically, as a Kuhnian paradigmatic case.

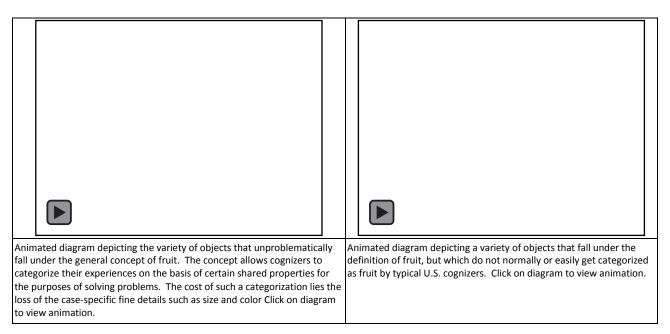
By analogy, I suggest that engaging in a more fine-grained analysis of the elements of typical paradigmatic cases and the ways in which scientists elaborate upon and extrapolate from these cases to develop paradigms can result in significant insight into the formation, presentation, and further development of scientific treatments of specific domains. One need not seek these insights through the formulation of ill-conceived universal and exceptionless prescriptions for scientific practice. One merely needs to take note of the more prominent underlying elements of such cases and the general patterns that tend to emerge as cases coalesce and domains become codified.

Specifically, I analyze paradigms and scientific domains in terms of eight elements. The treatment here remains at the level of relatively superficial generalizations. The analysis, nevertheless, proves sufficiently rich to provide structure for the rest of the chapters in this text.

## 1.3.a Eight Elements of Paradigms

The first three elements of paradigms discussed in this chapter provide the basic elements of theories. I'll start by discussing categorization—the basic classes into which theorist sort the raw phenomena of a domain. One of the most important differences between ontological frameworks and theoretical treatments of phenomena in a more developed science lies in the systematic, intersubjective, and repeatable methods theorists employ to tie their categorizations to the phenomena. I use the term operationalization to describe the techniques used to categorize and measure elements of a domain. This chapter and lectures introduces the characterization of theories as collections of models. Simply stated a model is a structured relationship between a set of categories. Models represent the phenomena in the domain in so far as their categories and relationships between those categories systematically map onto the objects, properties, or events in the domain. Operationalizations play a crucial role in establishing that mapping. Experimental traditions further elaborate and verify the integrity of that mapping through careful, systematic, and intersubjectively verifiable tests of that mapping. Data accumulation help to establish the limits of such models, identify potential problems for models, and identify potential elaborations and alternative formulations for models. The remainder of this section spells out each of these elements. The next section illustrates these elements in operation through the example of the development of the contemporary domain of oscillating systems theory from the ideal pendulum law.

**1.3.a.1** Categorization: Researchers develop categorizations of phenomena allowing them to differentiate types of objects, events, properties, and relations. Categorizations gain generality by divorcing themselves from the fine details of individual cases to unite many instances under a common type (usually on the basis of their shared similarities in certain respects and to certain degrees). People tend to focus upon the representational content of categories. This focus leads to insufficient appreciation for the trade-off between the loss of case-specific information and cognitive economy of commonality essential to the categorization process. For instance, an apple and an olive both fall under the common and very useful biological categorization of fruit: Each is the seed-bearing structure of flowering plants formed from the ovules (female reproductive structure) after flowering and fertilization occurs. However, while many people consider black olives a savory treat, they show markedly less gustatory enthusiasm when presented with black apples--a fact lost in the biological distillation.



1.3.a.2 Operationalization: One important feature of ontological frameworks is the looseness of the ties between the categorizations, attributions, and relations of ontological frameworks to phenomena. That is, categorizations often exhibit significant vagueness when applied to phenomena; often ontological frameworks exhibit few if any systematic and/or intersubjective methods or rules for applying their categories to phenomena. As the next chapter and lectures emphasize, this looseness of fit between the categories, attributions, and relationships in ontological framework impedes progress by masking or minimizing problems within the framework. One significant example of such difficulties is labeled "tenuous dualism" in the next chapter and lecture. Thus, one of the more significant factors in the development of a scientific treatment of a domain occurs when researchers develop and refine operationalizations. Operationalizations are methods and tools that facilitate systematic and intersubjective categorizations of phenomena. Operationalizations work either qualitatively (yes/no, in/out categorizations) or quantitatively (measurement, increasing/decreasing categorizations). The increased systematic, rigorous, and intersubjective categorizations of operationalizations render both the categories and the theoretic models of a burgeoning science much less vague and much more testable. Qualitative operationalizations allow theorists to apply categorizations in a systematic and intersubjective fashion. Quantitative operationalizations likewise allow for the quantification of categories integral to many theoretic models. For example, biologists apply a qualitative

operationalization process to potential fruits, sorting them into fruits and non-fruits. In contrast, psychologists employ a quantitative operationalization when discussing the typicality rating of fruits—a numerical index of how well a given item matches people's concepts.

**1.3.a.3** Theoretic Models: Researchers analyze phenomena into component categories, utilizing these categories to formulate dynamical and attributional models (i.e., theories). Dynamical models, such as the ideal pendulum law discussed below, depict the dynamical evolution of phenomena through time as a function of the interrelationship of component elements. Attributional models either assign attributes to phenomena or articulate the underlying basis for various attributes of phenomena. "Water is  $H_2O$ " represents one sort of attributional theory. As I employ it in this text, the notion of an attributional model serves as an umbrella for a collection of models discussed in greater detail by Cummins.<sup>1</sup>

Like the categorizations from which theorists construct them, theoretic models involve trade-offs. Specifically, models must often trade expressive detail, complexity, and predictive power for tractability, i.e., trade expressive detail, complexity, and predictive power for models with which one can reliably calculate answers given one's computational resources. To paraphrase Ronald Giere; the most accurate and complete model of the world is the world itself—good luck getting it inside your head and manipulating it in all and only the ways you need to make predictions in a timely fashion.

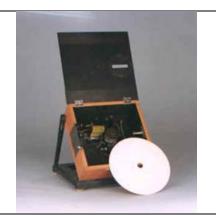
A commonly known and historically infamous example of the trade-off between complexity and expressive detail in theoretic models is known as the three-body problem. In Newtonian physics the problem of calculating the motions of three or more bodies from initial data on their positions, masses, and velocities proves non-computable. Specifically, the solution would require solving nine differential equations simultaneously. During the period between 1687, when Newton first discusses the problem, and 1910 physicists and mathematicians develop a number of approximate solutions—usually by introducing simplifying assumptions such as restricting the motion to two dimensions. Interested students can read about the three-body problem on Wikipedia <sup>92</sup> and Scholarpedia. <sup>93</sup>

**1.3.a.4** Data Accumulation: Developing a sufficiently broad and accurate understanding of the actual behavior of the elements of a domain proves integral to categorization and theorizing. Researchers often develop relatively systematic collections of raw observational data describing phenomena in a prospective domain. Such collections can identify regularities at a superficial level without detailing and relating their component elements, or these collections can specify more elaborate analyses of the phenomena. For instance, Galileo reports a superficial regularity when he notes that the motion of pendulums exhibits isochronism. **Isochronism** asserts that periodicity remains constant through increases in swing amplitudes and proves crucial to the original adoption of pendulums for clocks. Later Christiaan Huygens refines Galileo's observations, noting that increases in periodicity become significant when swing amplitude exceeds about 4° to 6°. Cummins calls such collections of data "effects." To explain the phenomena categorized as pendulum periodicity Galileo also introduces a more elaborate analysis of pendulum motion when he formulates the ideal pendulum law discussed below.

Researchers collect and organize data both in their initial investigations and during the course of the further development and elaboration of a domain within a particular paradigm. While this data provides grist for the theoretic enterprise, it also serves to correct misconceptions embodied in ordinary experience, to fill in holes

where ordinary experience rarely strays, and provide regularities for which researchers can seek attributional and dynamical models. Cognitive science provides a profoundly salient and seemingly endless source of examples of how poorly human beings understand how the world actually works. No one I've ever met, for instance, has ever mentioned how ordinary experience has led them to notice the role that pupil dilation plays in their judgments of facial attractiveness. Nor do they reflect upon how the role of pupil dilation has changed as they have grown older. Indeed, so profound is human ignorance of how humans actually work that Cummins has only half-jokingly described psychology as more of a gallery of effects than a set of theories.<sup>2</sup>

**1.3.a.5** Developing Experimental Traditions: Researchers likewise develop or adapt experimental methodologies and apparatus to test their models. The set of accepted experimental designs and apparatus constitute an experimental tradition. For example, in order to study memory the Hungarian psychologist <a href="Pál (Paul) Ranschburg">Pál (Paul) Ranschburg</a> creates a device, "The Ranschburg Memory Device," to allow for the controlled visual







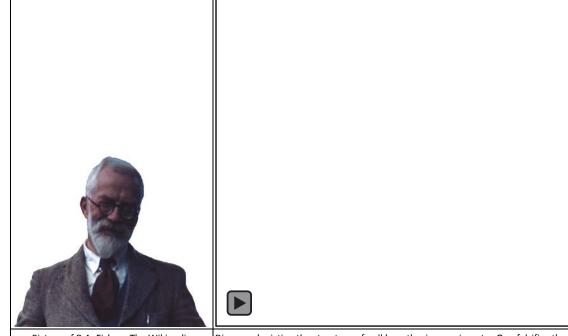
(Left) To study memory Hungarian psychologist Pál (Paul) Ranschburg developed "The Ranschburg Memory Device" in the early 1900s. The device consists of a box containing a rotating disk visible through a slot in the top. Experimenters can use the box to visually present a successive series of stimuli such as letters, words, numbers, symbols etc. at a controlled rate of presentation. The experimenter draws the stimuli on the disk and uses the box's motor to control the rate of presentation of stimuli through the slot. From: University of Toronto (Right) The tachistoscope allows researchers to present images for a specific and controllable time period. The inventor of the tachistoscope remains unclear, though historians usually attribute the first general description of the device to the psychologist A.W. Volkmann in 1859. From: National Taiwan University and Recycled reads.

presentation of symbols, numbers, words, etc. around 1900. Similarly, to study perception researchers create the tachistoscope <sup>96</sup> around the mid-1850s.

One can find another example of the development and utilization of experimental methodologies in the introduction and proliferation of null hypothesis testing. Null hypothesis testing seeks to accumulate evidence for a target hypothesis,  $h_t$ , by testing and rejecting the null hypothesis,  $h_0$ . In a fascinating bit of irony, the hypothesis that theorists test, the null hypothesis,  $h_0$ , hypothesizes that the suspected causal factor has **no effect**. Indeed, the power of null hypothesis design lies in the fact that—unlike the likelihood of the target hypothesis,  $h_t$ --theorists can quite easily calculate the probability that the causal factor in  $h_t$  affects the outcome given the null hypothesis,  $h_0$ --It is zero!!

One gathers evidence for the target hypothesis (not the null) by gaining negative evidence for (i.e. falsifying) the null hypothesis. Experimenters compare two groups, the control group, lacking elements of  $h_t$ , and the experimental manipulation group, where elements of  $h_t$  are ubiquitous. The null hypothesis,  $h_0$ , merely asserts that any difference between the experimental manipulation group and the control group will not exceed differences due to chance alone.

The widespread use of what Fisher eventually calls null hypothesis testing begins in R. A. Fisher's research in agriculture and genetics. Fisher designs the technique to solve a specific problem in experimental design—how to determine the probability of an effect given a particular problem. For example, how does one estimate the likelihood of a hypothesis like magnetic forces could bend light? The null hypothesis design solves this problem by creating and testing a hypothesis, the null hypothesis, for which one can easily determine the probability. Fisher presents and defends the technique in "On the Mathematical Foundations of Theoretical Statistics." As a result of the less technical presentations in his books, *Statistical Methods for Research Workers* and *The Design of Experiments*, researchers adapt the technique (for better or worse) across a wide range of scientific disciplines. The chapter on psychology discusses the rise of the experimental tradition in psychology, noting the development and adaptation of many other experimental techniques such as measuring reaction times. The development of a robust and creative experimental tradition plays an important part of the development and elaboration of any paradigm.

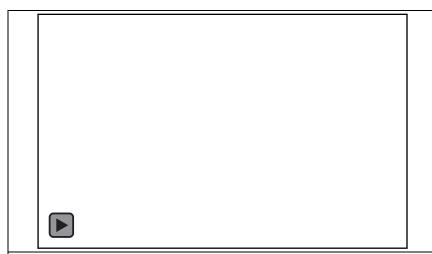


Picture of R.A. Fisher. The Wikipedia biography of R. A. Fisher can be found here: http://en.wikipedia.org/wiki/Ronald\_Fisher The Wikipedia page on null hypothesis testing can be found here: http://en.wikipedia.org/wiki/Null\_hypothesis

Diagram depicting the structure of null hypothesis experiments. One falsifies the null hypothesis (and gains evidence for the target hypothesis) if the introduction of the suspected causal factor into otherwise identical circumstances alters the outcome by more than the margin of error (generally .05). Imagine creating two cloned Earths. These Earths remain identical in every way except one: On one Earth, E<sub>1</sub>, the suspected causal factor proves completely absent (call this the control planet), on the other Earth, E<sub>2</sub>, the suspected causal factor is omnipresent (call this the experimental manipulation planet). If the suspected causal factor is not a causal factor (the null hypothesis), samples from the experimental manipulation world should fall within the margin of error of samples from the control world. If the suspected causal factor is a causal factor, samples from the experimental manipulation world should differ from samples from the control world at a rate greater than the margin of error. Click on diagram to view animation.

**1.3.a.6** Explanatory Schema: Researchers elaborate and refine one or more explanatory schemas in developing scientific treatments of a given domain. These schemas consist of outlines or strategies for treating new or problematic aspects of the phenomena in the domain that researchers derived from the salient paradigmatic cases. For instance, biologists use the general schema of evolution to explain the differences in the expression of traits across entire species. Evolution provides a general framework for depicting differences and mechanisms by which such traits appear and propagate. Similarly, the evolution of early treatments of pendulum motion sought to understand that motion through the relationship between periodicity, armlength, and gravitational acceleration.

**1.3.a.7** Generalized Solution Strategies: Researchers do not just develop theoretic models and experimental traditions. Researchers also develop generalized solution strategies as part of their treatments of scientific domains. Generalized solution strategies consist of techniques for adapting and manipulating their theoretical models, allowing scientists to utilize those models to generate predictions, retrodictions, manipulations, and explanations across a wider range of applications. For example, in order to explain how gravity moves objects down inclined planes [movement both along the downward direction (i.e., vertical motion) and along the direction orthogonal to the downward force of gravity (i.e., horizontal movement)] physicists introduce the generalized solution strategy of component forces.



Animated diagram depicting an object on an inclined plane. One explains the motion of the object on the plane by analyzing the force of gravity (m=mass, g=gravitational attraction,  $\theta$ = the angle of inclination) into component forces. One multiplies the force of gravity, mg, by the cosine of the angle,  $\theta$ , of planar inclination to generate the normal force, N. One then treats the normal force as the force exerted directly down (perpendicular to) on the surface of the inclined plane. So, one uses the normal force to determine frictional resistance generated between the surface of the plane and the surface of the object. One multiplies the force of gravity, mg, by the sine of the angle,  $\theta$ , of planar inclination to determine the force parallel to the surface of the inclined plane, f, which one uses to calculate the acceleration (and velocity) of the object as it moves down the plane. Click diagram to view animation.

Physicists do not suppose that gravity acts along the angle indicated by the force line labeled  $mg cos\theta$  and N--also often called the normal force and sometimes symbolized as F<sub>n</sub>. Physicists define the normal force as the force line perpendicular to the plane's inclined surface. Likewise, physicists do not suppose that gravity acts along the angle indicated by the force line labeled f and  $mg sin\theta$ . The force line, f, parallel to the surface of the plane represents the force acting to move the object along the inclined surface, usually symbolized as f and mg sinθ. Instead, gravity exerts a force directly downward along the line indicated by mg which also represents the object's weight in the gravitational field.

**1.3.a.8** Accepted Partial-Potential Models & Success Criteria: An integral aspect of scientific practice as I depict it involves disabusing oneself of the false view that science formulates universal, exceptionless laws expressing (or that will eventually express) the exact and complete nature of the phenomena subsumed by those laws. Theories often have exceptions, scope limits, and perhaps most importantly; theories have no specific privileged formulation. As Frederick Suppe notes, <sup>14</sup>

...the heart of a theory is an extralinguistic *theory structure*. Theory structures variously are characterized as set theoretic predicates (Suppes and Sneed), state spaces (Beth and van Fraassen), and relational systems (Suppe). Regardless which sort of mathematical entity the theory structures are identified with, they do pretty much the same thing-they specify the admissible behaviors of state transition systems. (p.4)

Thus, theories use a variety of mediums to symbolize or represent the categories and relationships between those categories through the target phenomena are modeled. Theoretic models utilize some representational medium(s) (symbols, numbers, diagrams, etc.). The particular medium with which scientists formulate theoretic models shapes to some extent the information and operations one can utilize in formulating and manipulating those theoretic models. This last point has an excellent illustration later in this chapter during

the discussion of the idealized pendulum law. For now consider the following, very simple example: Suppose one represents pi as the symbol,  $\pi$ , in the formula for the circumference of a circle:  $C = d \cdot \pi$  (where C =circumference, d = diameter, and  $\pi = pi$ ). One cannot, given one's choice of representational medium for pi, calculate circumferences expressed exclusively as explicitly represented decimal approximations of the circle's circumference. For instance, if the diameter equals 5 inches, then one's calculated circumference equals  $5\pi$ inches. On the other hand, if one uses a decimal approximation of pi, like 3.14159265, then one can calculate circumferences expressed exclusively as explicitly represented decimal approximations, e.x.,  $5 \cdot 3.14159265 =$ 15.70796325 inches. Using  $\pi$  to represent pi makes representing and calculating the circumference easy enough that people can calculate the circumference in their head—without the aid of a calculator or paper and pencil. However, one's answer does not really help one determine, say, how long a piece of string one needs in order to wrap string around the outside of the circle. Representing pi as a decimal approximation like 3.14159265 allows one to calculate the circumference if a circle with great precision. However, most people would have great difficulty calculating the circle's circumference without a calculator or paper and pencil. Similarly, the specific approximation, 3.14159265, likely provides a greater potential for accuracy in calculating the circumference than the average accuracy of the average person's measurement skills. In other words, one may determine that the circumference equals 15.70796325 inches, but few people could measure and cut a piece of string to exactly that length.

As a result of the above considerations, one must conclude that theories necessarily depict the world through the selective lens of abstraction and the specific properties of the chosen representational medium. Categorizations gain generality by divorcing themselves from the fine details of individual cases to unite many instances under a common type. Choice of representational medium and the elements included in the theoretic model introduces tradeoffs between complexity and predictive power and tractability (where a tractable model is one with which one can calculate answers, i.e., use).

Thus, scientific practice inherently involves tradeoffs between factors like details, complexity, predictive power, and representational medium. Scientific progress and consensus, as a result, can only occur against a backdrop of norms for success. Researchers must converge upon generally acceptable norms for determining the adequacy of predications and explanations of phenomena within the domain. Part of the formulation of acceptable norms involves reaching rough agreement as to the appropriate uses of models for solving problems. I call these acceptable uses partial potential models. For instance, the ideal pendulum law abstracts from a number of relevant factors to capture pendulum motion. One important factor ignored by the ideal pendulum law is frictional resistance at the arm-pivot of pendulums. For this reason, scientists agree that all acceptable partial potential models for the ideal pendulum law must have negligible frictional resistance at the arm pivot. The law, as a result, applies only to those instances that fall within the scope of acceptable partial potential models. As the discussion about pendulums below illustrates, people must also reach a general agreement regarding the standards for success of a given set of theoretic models together with the categorizations and operationalizations that connect those models to the phenomena.

#### 1.4 Applying the Eight Elements to the Development of Oscillating Systems Theory

The remainder of this chapter utilizes the above-mentioned notions to outline the general processes whereby researchers (A) come to view a set of phenomena as constituting a scientific domain and (B) develop and refine treatments of that domain. The process begins when researchers identify domains of interest--that is, domains in which the phenomena appear to exhibit both a unity and regularity. The unity and regularity

needed to highlight a domain for potential scientific treatment can manifest itself in the dynamical interactions and/or development of domain elements over time or in the properties exhibited by objects, events, or relations within the domain. While unified and regular dynamic phenomena give rise to dynamical models, attributional models capture unity or regularity manifested as attributes of objects, events, or relations within the domain.

For instance, this chapter uses the ideal pendulum law, a dynamical theory of pendulum motion, as its central illustrative example. Part of that ideal pendulum law is the constant, g. This constant represents part of a unified theory of gravitational acceleration—namely the uniform acceleration of matter within the Earth's gravitational field. As students might recall, this theory assigns a constant value to gravitational acceleration;  $g = 9.8 \text{m/s}^2$ . In short, g represents an attributional model of gravitational acceleration—a theory intended to capture the disposition of objects to accelerate when unrestrained in the Earth's gravitational field.

Researchers make further progress by collecting data about the phenomena in the domain, developing categorizations of phenomena that appear to capture unity and that facilitate the expression of regularities in the form of theoretic models. The testing of these categorizations, dynamical theories, and attributional theories both requires and facilitates the establishment of a robust experimental tradition within the domain. At this point, it becomes possible for a paradigmatic treatment, in Kuhn's first sense, to emerge. As researchers and research organizes around one or two paradigmatic treatments, a general explanatory framework emerges that serves to structure further research. More standardized treatments of the domain structured around the paradigms emerge. These treatments include both pseudo-physical models to aid in conceptualizing the world as portrayed in the theory and a set of generalized solution strategies for adapting and further articulating the central theoretic models to encompass novel instances and important variants of the central cases. Finally, the domain may expand or contract as further instances come under the explanatory schema or further differentiations between types of phenomena occur.

#### 1.4.a Theories as Models: The Ideal Pendulum Law

The last two paragraphs might seem like word salad to many students, so let us turn to a more concrete treatment. Consider a simple theory with which many students have some familiarity, the ideal pendulum law: In English, the law asserts that the periodicity (the time it takes the bob of a pendulum to swing from one extreme of its arc and back again) equals two *pi* times the square root of the length of the pendulum arm divided by the acceleration due to gravity. This treatment of the pendulum dates to Galileo's investigations into pendulum motion around 1602. Galileo discusses his findings regarding pendulums and offers further speculations in a 1602 letter to <u>Guido Ubaldo dal Monte</u>. <sup>100</sup> (pp.97-100) When he publishes <u>Dialoque on the Two Chief World Systems</u> in 1632, Galileo reports discovering that periodicity varies in a proportional fashion to the square root of length of the arm. <sup>101</sup> That is, increasing arm length increases periodicity. In addition to relating arm length to periodicity, Galileo reports the independence of periodicity from both the mass of the bob and (approximately) the amplitude of the swing. In other words, neither increasing the mass of the bob,

$$P = 2\pi \sqrt{\frac{L}{g}}$$
 frictionless pivot



(Top left) The standard formulation of the ideal pendulum law; P = periodicity, L = length of the arm pivot, and g = acceleration due to local gravitational attraction. (Left middle) Galileo's drawing of his experimental pendulums from *Dialogues on the Two Chief World Systems* (figure 21). (Top right) A schematic labeled diagram depicting the ideal pendulum. Image from Wikipedia

nor increasing the swing size affects periodicity. Theorists call the approximate constancy of periodicity across different swing amplitudes isochronism, and this feature of pendulums, though not as robust as Galileo reports, proves extremely significant as well as consistent with his general understanding of the domain of

terrestrial motion. Galileo's findings provide a theory for pendulum motion in the form of the equation in the table below (top left). But what does it mean to say that scientific theories like the ideal pendulum law are models? What does it mean to say that a mathematical equation models a pendulum? Focusing upon the equation itself, one can observe two important features of theories. First, theories assert relationships between types or categories. Thus, the ideal pendulum law relates periodicity (a type of time period) to arm length (type of object property) and local gravitation acceleration (a type of force). Specifically, the categories that theories employ allow theorists to depict phenomena through an analysis of the phenomena into more elemental components and their relationships. Thus, the ideal pendulum law analyzes the movements of pendulums by expressing the pattern of those movements in terms of the relationships between component elements of that movement. The ideal pendulum law categorizes pendulum movement in terms of periodicity--the time it takes the bob to move from one end of the swing arc to the other and return. Periodicity allows theorists analyze the continuous movements of pendulums into discrete component movements. These component movements (i.e., periods) also divide the continuous temporal evolution of the pendulum into discrete temporal components. Theorists then depict the motion of pendulums in terms of the relationship between periodicity, on one hand, and two other component elements of the phenomena-arm length, and uniform gravitational acceleration. The equivalence relation between categories in the mathematical structure of the ideal pendulum law mirrors the structural relationship between time, motion, length, and acceleration in pendulums.

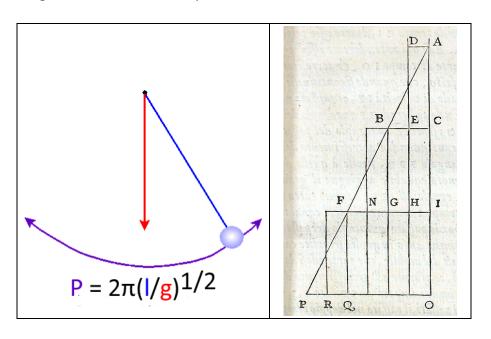
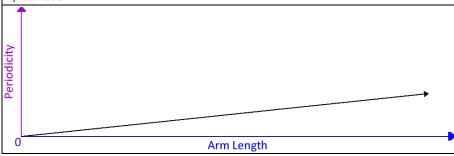


Diagram (top left) depicting the relationship between elements of pendulum motion utilizing the categories of the dynamic theoretic model called the ideal pendulum law. Diagram (top right) from Galileo's *Dialogues on the Two Chief World Systems*. (figure 47) Galileo uses this diagram to represent the relationships between time, velocity, acceleration, and distance traveled. The horizontal lines BC, FI, and PQ represent velocities. The vertical lines AC, CI, and IO represent times. Thus, the lines AB, BF, and FP represent the rate of acceleration and the area of triangles ABC, AFI, and APO represent the distance traveled. Diagram (below) depicting the ideal pendulum law using a state-space representation.



The relationship between features of a model and features of the domain need not find expression in mathematics; one can formulate the theory in other representational mediums. For instance, when I first mentioned the pendulum law, I expressed the relationship in English. I've also expressed it as an equation (as the color coded equation in the top left of the table on the right). I express the relationship as a state-space (as in

the diagram in the bottom panel of the table) and as volumetric relationship between geometric figures (in the diagram on the top right of the table). What, then, makes something an expression of a theory? I suggest that a model (representational structure) expresses a theory when elements of the model's structure systematically map onto the elements of the phenomena in such a way that the model's structure, as interpreted through the mapping, expresses the theory's insight into the phenomena. For instance, a state-space like the one depicted below expresses the ideal pendulum theory because of the following mapping: (1) The horizontal (blue line) direction represents the length of the pendulum arm increasing from the origin of zero (far left). (2) The vertical direction (purple line) represents the duration of the period increasing from the origin of zero (bottom left). (3) The black line represents the values for each period corresponding to each arm length. This variability in the formulation of theories is crucial to understanding scientific theories in that it deemphasizes the role of some or other canonical formulation (e.x. an exceptionless universal statement) in understanding theories in favor of their actual function in the scientific enterprise. Indeed, Galileo draws extensively upon geometry both in formulating and in proving various theories in *Dialogue on the Dialogues on the Two Chief World Systems*. <sup>101</sup>

The second aspect of theories that the pendulum law equation brings into focus is that theories model phenomena only in certain respects and to certain degrees. Indeed, the ideal pendulum law only models certain features of pendulums (respects). Some features of real-world pendulums neglected by the law include properties irrelevant to periodicity like color, material composition, ownership, the mass of the bob, etc.. Still other features of real-world pendulums that the theory fails to model prove relevant to periodicity (e.x., frictional and medium resistance). As a result, the relationship between periodicity, arm-length, and gravitational acceleration expressed in the ideal pendulum law only approximates the behavior of real-world pendulums. That is, the relationship asserted in the ideal pendulum law only models real-world pendulums to a certain degree of accuracy.

Specifically, the ideal pendulum makes simplifying assumptions about the pendulums it describes. These simplifying assumptions allow for the formulation of equations tailored to specific uses and interests, equations that prove computationally tractable (i.e., the equations have relatively straightforward and calculable solutions). However, the benefits of these simplifying assumptions come at a cost. Should the real-world pendulum system violate these assumptions to a significant degree, systematic inaccuracies in

prediction or, at minimum, the potential for systematic inaccuracies in prediction can result from the use of the ideal pendulum law.

Indeed, the ideal pendulum law assumes a massless and perfectly rigid pendulum arm. Likewise, it assumes zero resistance from the medium (the air through which the pendulum travels) and zero frictional resistance at the arm pivot. It assumes that one can treat the mass of the bob as a point-mass because the bob's mass is evenly distributed. Finally, it assumes that the amplitude of the swing does not affect periodicity. Strictly speaking, none of these assumptions holds true for real pendulums. Thus, philosophers like Nancy Cartwright assert that<sup>5, 6</sup>

In modern physics, ..., phenomenological laws [laws scientists formulate specifically to capture explicitly observed data] are meant to describe, and they often succeed reasonably well. But fundamental equations [laws] are meant to explain, and paradoxically enough the cost of explanatory power is descriptive adequacy. Really powerful explanatory laws of the sort found in theoretical physics do not state the truth. ... I will argue that the accounts they [fundamental laws] give are generally not true, patently not true by the same practical standards that admit an indefinite number of commonplace phenomenological laws. We have detailed expertise for testing the claim of physics about what happens in concrete situations. When we look to the real implications of our fundamental laws, they do not meet these ordinary standards. Realists are inclined to believe that if theoretical laws are false and inaccurate, then phenomenological laws are more so. I urge just the reverse. When it comes to the test, fundamental laws are far worse off than the phenomenological laws they are supposed to explain. (p.3)

Because of the implicit simplifying assumptions built into the ideal pendulum law, scientists also have conventions regarding its use. Specifically, one can use the ideal pendulum law to predict the behavior of pendulums within a tolerable margin of error for relatively small swing amplitudes in so far as the arm mass, fictional resistance, and medium resistance prove negligible. The number of pendulums having negligible frictional and medium resistance proves relatively small in the real world. Discussions of pendulums, therefore, often move from the ideal pendulum law to other models that include frictional and/or medium resistance—usually called damped harmonic motion. These models tend not to differentiate between resistance due to the medium and resistance due to friction at the arm pivot. While combining these two forces again results in a less accurate theoretical depiction of pendulums, it represents a trade-off of descriptive accuracy for ease of use—accuracy for simplicity.

But why, then, does the ideal pendulum law ignore frictional resistance to begin with? I suggest three reasons. First, Galileo conducts the original studies using pendulums consisting of weights suspended by strings, thereby minimizing frictional resistance. Second, Galileo's discovery of uniform gravitational acceleration (in his famous, though possibly fictional, experiments in which he drops objects from the leaning tower of Pisa) no doubt shapes Galileo's theoretical perspective in investigating pendulums. Third, since frictional and medium resistance increases proportionally with velocity, these models prove much more complicated. Indeed, the standard treatments of damped pendulums describe pendulums using different categorizations. These models move from a dynamical analysis of pendulum motion in terms of periodicity to analyses in terms of position, velocity, and time. The models likewise employ the mathematics of trigonometry and calculus, neither of which is available to Galileo. In short, these models introduce alternative categorizations and alternative inter-category relationships in order to capture the dynamic evolution of pendulum motion. The theoretic treatment of pendulums remains unified in that the models

retain the basic insight of the ideal pendulum law—the relationship between arm length and oscillatory motion. Likewise, periodicity remains definable and calculable within the alternative categorizations, though the calculations become much more complex.

Similarly, when swing amplitude exceeds about 4° to 6°, isochronism no longer proves viable. Recall, isochronism asserts that periodicity remains constant through increases in swing amplitudes, and proves crucial to the original adoption of pendulums for clocks. In point of fact, as swing amplitudes increase periodicity likewise increases. Unlike frictional resistance, violations of isochronism for larger swing amplitudes represent a real empirical discovery. Christiaan Huygens' <u>Horologium Oscillatorium sive de motu Pendulorum</u> (1673) represents the first systematic demonstration of periodicity variation due to swing amplitude. Modifications of the pendulum model to include swing amplitude prove equally complicated. For illustrative purposes I include these more complicated models without further comment.

$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\sin\theta = 0$$

 $\frac{d^2x}{dt^2} + 2\zeta\omega_0 \frac{dx}{dt} + \omega_0^2 = 0$ 

**Above:** The equation for calculating the motion of a pendulum for all swing amplitudes. For small swing amplitudes the equation reduces to the equation for simple harmonic motion:  $\theta(t) = \theta_0 \cos(2\pi t/T)$  where  $\theta$  is the angular displacement in radians, t = time, and T equals periodicity.

**Above:** Pendulum law taking frictional resistance into account. Here x represents the current position of the bob,  $\zeta$  is dampening ratio and  $\omega_0$  is the undamped angular frequency. For small damping one generally adopts the *Ansatz* equation:  $x(t) = Be^{-\omega_\gamma t} \cos(\omega' t)$ .

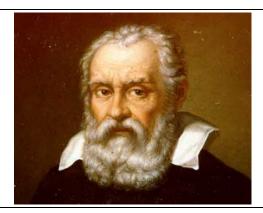
$$\frac{\Delta T}{T_0} = \sum_{n=1}^{\infty} \left( \frac{2n!}{2^{2\pi} (n!)^2} \right)^2 \sin^{2n} \left( \frac{\theta_0}{2} \right)$$

**Left:** In order to determine the real periodicity of a pendulum one must calculate both the ideal periodicity,  $T_0$ , and the difference between the ideal periodicity and the real periodicity,  $\Delta T$ , usually called the *circular error*. One then calculates the real periodicity,  $T_0$ , thusly:  $T_0 = T_0 + \Delta T = T_0 (1 + \Delta T / T_0)$ . Above is the complete elliptic integral of the first kind for the Jacobian elliptic sine function used to determine the increased value for periodicity for any angular displacement (given in radians);  $\theta = 0$  the maximum angular displacement in radians,  $\Delta T = 0$  the difference between the ideal periodicity and the real periodicity, usually called the *circular error*, and  $T_0 = 0$  the ideal periodicity.

The modifications of the basic pendulum law provide two more insights into the nature of theories. First, theories are collections of models, each representing extensions or modifications of the central model or insight for specific situations or applications. Second, the development of theories in the form of additional models is driven by both the nature of the phenomena itself as well as the needs and interests of theorists. For instance, the elaborations of the ideal pendulum law come in part because of the interests of scientists—in this case keeping time. Indeed, Galileo himself uses pendulums as timing devices in simple metronomes. One of his friends, Santorio Santorio, creates a device, the *pulsilogium*, to time the pulse of his patients. Towards the end of his life Galileo even develops a design for the pendulum clock. Christiaan Huygens built the first known pendulum clock in 1656, and modifies his clocks in light of his discovery of the failure of isochronism for large swing amplitudes. Pendulum clocks initially increase the accuracy of clocks from approximately 15 minutes a day to 15 seconds a day. Modifications of pendulum clocks increase accuracy so that US National Bureau of Standards utilizes pendulum clocks as late as 1929.



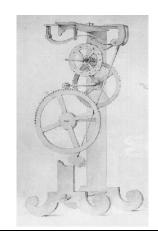
Portrait of Santorio Santorio 1561–1636



Portrait of Galileo February 15, 1564 - January 8, 1642



Portrait of Christiaan Huygens April 14, 1629 – July 8, 1695



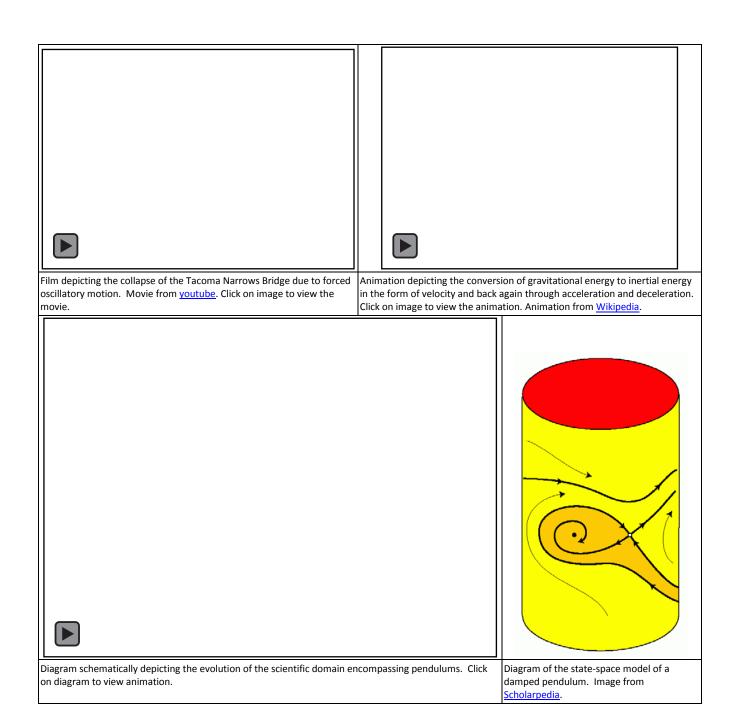
Galileo's conception for the pendulum clock. Image from Wikipedia.



Huygens' clock together with a copy of *Horologium Oscillatorium sive de motu Pendulorum*. Image from Wikipedia.

Not surprisingly, the additional models for pendulum motion discussed above do not exhaust the models. Indeed, researchers also have an interest in the potential energy contained in the pendulum (energy stored in the system) at various points in its swing since one can convert potential energy into to mechanical energy to drive machines other than clocks. The interest in the energy of pendulum systems spawns diverse treatments. Moreover, if one opens a physics textbook today, one does not find a chapter on pendulums. Rather the chapters discussing pendulums, like the domain itself, have moved from generalized harmonic systems to oscillatory systems.

Contemporary categorizations of systems like pendulums classify these systems according to several properties. First, periodic motion occurs when an object's motion exhibits repetitive patterns. In the case of pendulums, the bob moves along the swing arc over and over. Harmonic motion refers to periodic motion in which roughly proportional displacing and restoring forces govern the motion.



In the case of pendulums, the displacing force, gravity, accelerates the bob during the downward arc of swing, converting potential gravitational energy into inertial energy in the form of velocity. At the bottom of the swing, all energy in the system takes the form of inertial velocity driving the bob through the upward arc of the swing. During the upward arc, the inertial energy reconverts into potential gravitational energy through deceleration, and so on. Thus, like all harmonic systems, the pendulum moves about an equilibrium position—a place in which the restoring and displacing forces normally equal one another. Moving a harmonic system from the equilibrium position introduces an unbalanced relationship between displacing and restoring forces, causing the periodic motion about the equilibrium position—i.e., oscillations.

Of course, not all oscillatory systems have single, stable equilibrium positions, thus simple harmonic motion represents an important subset of oscillatory motions. Other important oscillatory motions include damped, driven or forced, and coupled oscillatory motions. The characteristic systems in such chapters include

weighted spring systems, pendulums, and rotating bars. In the box above, the movie (top left) shows forced oscillatory motions destroying the Tacoma Narrows Bridge. The animated diagram (top right) illustrates the harmonic motion of a pendulum. The animated diagram (bottom left) depicts the changing structure of pendulum domain. The diagram (bottom right) depicts the state-space model of a damped pendulum.

# 1.4a Categorization

Though Galileo formulates what we now call the ideal pendulum law, his interest in pendulums comes primarily from his interest in gravitational acceleration. Galileo famously articulates the principle of universal gravitational acceleration. Prior to Galileo falling objects were thought to fall a differential rates corresponding to their weight. Galileo performed (or claimed to have performed) a number of experiments demonstrating uniform acceleration. Galileo likely did not drop balls from the leaning tower of Pisa. He did, however, perform careful experiments showing that balls of different weights accelerated at the same rate when rolled down inclined planes. Finally, Galileo performed experiments using pendulums (weights of various sizes suspended on strings) that seemed to show that these weights accelerated at a uniform rate through their swings. For Galileo, then, pendulums attracted his interest initially as applications of the principle of uniform gravitational acceleration. Thus, the ideal pendulum law invokes three categorizations: periodicity, arm length, and g, the constant for uniform gravitational acceleration. As noted above, the latter treatments of pendulum motion involve quite different categorizations such as position, velocity, and time frictional coefficient, and angular displacement.

#### 1.4.c Operationalization

Measuring length poses little problem for Galileo, but operationalizing periodicity and determining the value of g both require accurate time measurements. Galileo used his own pulse as an initial timing device when he first notices pendulum swing, but for his experimental work Galileo uses a water clock. To wit:  $^{101}$ 

For the measurement of time, we employed a large vessel of water placed in an elevated position; to the bottom of this vessel was soldered a pipe of small diameter giving a thin jet of water, which we collected in a small glass during the time of each descent, whether for the whole length of the channel or for a part of its length; the water thus collected was weighed, after each descent, on a very accurate balance; the differences and ratios of these weights gave us the differences and ratios of the times, and this with such accuracy that although the operation was repeated many, many times, there was no appreciable discrepancy in the results. (p.146)

Of course, the process here measures time in terms of relative weight and could be no more accurate than the scales, units for weight, and collection processes allows. Indeed, one reason Galileo seems to have used inclined planes and pendulums in his work likely lies in the fact that acceleration is easier to time accurately than the relatively quick vertical drop. Balls rolling down an inclined plane move much more slowly than they do when dropped from a tower. Using movement of balls rolling down inclined planes, Galileo determines that gravitational acceleration is a function of the square of the time. His estimate comes out to be about  $10.8 \, \text{m/s}^2$ , compared to our  $9.81 \, \text{m/s}^2$ .  $^{101}$ 

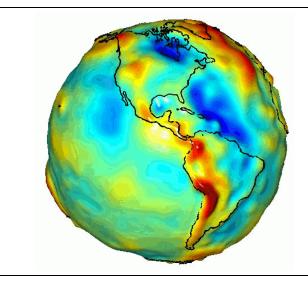
A piece of wooden moulding or scantling, about 12 cubits long, half a cubit wide, and three finger-breadths thick, was taken; on its edge was cut a channel a little more than one finger in breadth; having made this groove very straight, smooth, and polished, and having lined it with parchment, also as smooth and polished as possible, we rolled along it a hard, smooth, and very round bronze ball.

Having placed this[213] board in a sloping position, by lifting one end some one or two cubits above the other, we rolled the ball, as I was just saying, along the channel, noting, in a manner presently to be described, the time required to make the descent. We repeated this experiment more than once in order to measure the time with an accuracy such that the deviation between two observations never exceeded one-tenth of a pulse-beat. Having performed this operation and having assured ourselves of its reliability, we now rolled the ball only one-quarter the length of the channel; and having measured the time of its descent, we found it precisely one-half of the former. Next we tried other distances, comparing the time for the whole length with that for the half, or with that for two-thirds, or three-fourths, or indeed for any fraction; in such experiments, repeated a full hundred times, we always found that the spaces traversed were to each other as the squares of the times, and this was true for all inclinations of the plane, i.e., of the channel, along which we rolled the ball. We also observed that the times of descent, for various inclinations of the plane, bore to one another precisely that ratio which, as we shall see later, the Author had predicted and demonstrated for them. (p.146)

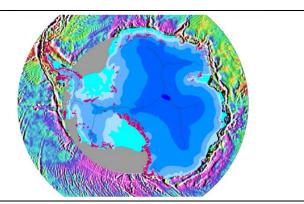
#### 1.4b Data Accumulation

The development of the treatment of oscillating systems like the pendulum illustrates another feature important to the development of scientific treatments of a domain—data collection. Prior to Galileo only a few people consider pendulums. Likewise, terrestrial motion generally has little real data upon which people can base categorizations and dynamical theories. Indeed, the theories of terrestrial motion prior to Galileo followed what people generally call "Aristotelian Physics." Aristotle was no stranger to terrestrial motion, and even seems to grasp inertia (though in what he considers a counterfactual situation: the void) in some of his writings<sup>102</sup> (Book IV, part 8, paragraph 7). However, Aristotle proposes that terrestrial motion occurs because each of the four elements (earth, air, fire, and water) moves toward their natural place in existence. Since these four elements compose all other materials each type of material moves toward the place representing the equilibrium between their component parts. Thus, rocks fall to earth because they are composed primarily of earth. There are obvious and striking difficulties with this view when confronted by the facts. For instance, why do arrows travel anywhere but straight into the ground once released as they too are primarily earth? Indeed, even today many people have little insight into very ordinary terrestrial motions. For example, if you accelerate in your car and you have a soda bottle laying on the seat with an air bubble in it, does the bubble move forward, backward, or does it remain in the same place? Many people think the bubble will move backward. In fact, the bubble will move forward.

Another important feature of scientific data collection lies in its systematic, replicable, and accurate nature. For instance, Galileo might not have dropped objects from the Tower of Pisa, but he did perform experiments in which he rolled balls down an inclined plane, he also performed a number of experiments with pendulums. Similarly, Aristotle himself seems to have determined that water is ten times denser than air (an estimate Galileo criticizes) based upon the relative speeds at which similarly shaped objects of differing weights fall



Earth's gravity measured by NASA's GRACE mission, showing deviations from the theoretical gravity of an idealized smooth Earth, the so called earth ellipsoid. Red shows the areas where gravity is stronger than the smooth, standard value, and blue reveals areas where gravity is weaker. Picture and caption from Wikipedia.



Gravity map of the Southern Ocean around the Antarctic continent. This gravity field was computed from sea-surface height measurements collected by the US Navy GEOSAT altimeter between March, 1985, and January, 1990. The high density GEOSAT Geodetic Mission data that lie south of 30 deg. S were declassified by the Navy in May of 1992 and contribute most of the fine-scale gravity information. The Antarctic continent itself is shaded in blue depending on the thickness of the ice sheet (blue shades in steps of 1000 m); light blue is shelf ice; gray lines are the major ice divides; pink spots are parts of the continent which are not covered by ice; gray areas have no data. Picture and caption from Wikipedia.

through water. Indeed, we now have a much larger and better body of data upon which to understand acceleration due to gravity in terrestrial motion. Just like data collection revealed difficulties with isochronism, data reveal uniform gravitational acceleration is not actually uniform. Acceleration varies depending upon latitude, altitude, and depth. Indeed, acceleration ranges from a value of  $9.780 \text{ m/s}^2$  at the equator to a value of  $9.832 \text{ m/s}^2$  at the Earth's poles.

# 1.4.e Explanatory Schemas

So, Galileo allows theorists understand a number of important features of terrestrial movement, which together provide a general schema for predicting, retrodicting, manipulating and explaining terrestrial motion. First, falling objects accelerate during falls--in contrast to the Aristotelian notion that such motion had a uniform velocity. Second, gravitational acceleration is uniform regardless of mass and material, and that rate of acceleration can be determined experimentally.

# 1.4.f Accepted Partial Potential Models and Success Criteria

Galileo's work proves quite useful for predicting, retrodicting, manipulating, and explaining terrestrial motion. For example, one could now calculate the trajectories of cannon balls and generate ballistics tables. However, the model assumes a number of factors influencing motion prove negligible. For example, frictional resistance and resistance of the medium both invalidate Galileo's models. Wind can affect ballistics calculations; rubber cubes will not move down an inclined plane at the same rate polished bronze balls roll down that plane; bronze balls fall slower in water than in air, and so on. The proper applications of specific models as well as the modification models for alternative cases becomes a large part of the work done by theorists after Galileo, as we have seen in the case of oscillating systems.

#### 1.5 Cognitive Science as Science

This section provides students with a preview of the overall picture of cognitive science and its treatment of various cognitive domains. The section's expository goal consists primarily in providing students with an overarching outline of the final picture that will emerge by the end of the introductory chapters and lectures.

## 1.5.a The Ontological Framework of Cognitive Science

Cognitive science operates under the general physicalistic framework of all the sciences, but it also has its own distinct set of categorizations and interrelationships. I adopt the monikers introduced by Robert Cummins in outlining these two theses: "Computational Theory of Cognition" (CTC) and the "Representational Theory of Intentionality" (RTI). Together the CTC and RTI provide the fundamental categorizations and relations within which most of cognitive science emerges. In its broadest statement, RTI asserts that mental states are about the world (have content) in virtue of a representation relation holding between the world and those states. The CTC holds that cognition is the computation of complex functions on such representational states, where computation consists in performing operations defined over the structure types of representational states. The combination of the CTC and the RTI create two general explanatory schemas that inform much of the theorizing in cognitive science. Cummins terms the schematic relationship between the cognitive processes

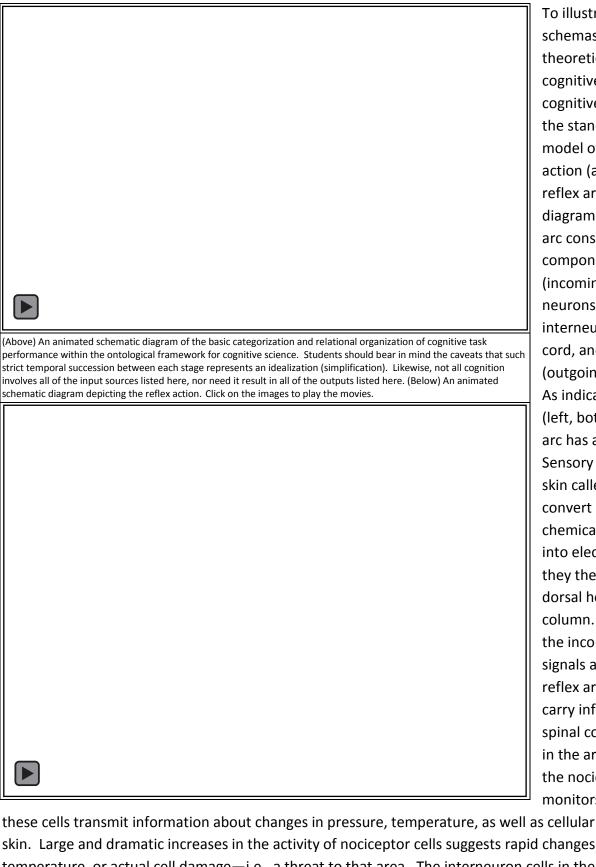


Animated schematic diagram of the tower bridge illustrating the basic categorization and relational organization of the CTC and RTI in the performance of a computation associated with a cognitive task within the ontological framework for cognitive science.

and the world the "Tower Bridge Model" of computation. <sup>4</sup> As illustrated in the animated diagram (left), the tower bridge portrays cognitive certain states, inputs to the system, as standing in the representation relationship with physical objects, properties, events, and/or relations in the world. Dynamic changes in the world result in new physical states. These dynamic changes in the world are mirrored by the dynamic processes that transform the initial representational state of the cognitive system to create a new, output, state. This output state also maps onto the new physical state generated by the dynamic changes in

the world. This similarity of structure between the states and dynamic changes in the world and in the cognitive system allows the cognitive system to model the world and its dynamical changes.

In addition to the tower bridge model of computation and representation, CTC/RTI also provides a general schema for cognitive processes. This three stage schema portrays cognizers as receiving input via sensory organs that transduce physical features of the world, retrieval of stored memories, and/or the output from a previous process. The cognitive process then operates upon the input states transforming them through computation so as to generate output states. Outputs then guide the system's behavioral response by becoming encoded as memories, by serving as outputs to another process, and/or by serving as motor response commands). This three stage schema for cognitive task performance serves to structure the formulation of models for predicting, retrodicting, manipulating, and explaining cognitive task performance.



To illustrate how these schemas structure theoretic models of various cognitive capacities in cognitive science consider the standard (simplified) model of the basic reflex action (also known as reflex arc) in the animated diagram below. The reflex arc consists of three main components; afferent (incoming) sensory neurons called nociceptors, interneurons in the spinal cord, and efferent (outgoing) motor neurons. As indicated in the diagram (left, bottom), the reflex arc has a direction. Sensory receptors in the skin called nociceptors convert mechanical and chemical signals in the skin into electrical signals that they then transmit into the dorsal horn of the spinal column. Theorists classify the incoming nociceptor signals as inputs to the reflex arc. These inputs carry information to the spinal cord about changes in the areas of the skin that the nociceptor cells monitors. Specifically,

these cells transmit information about changes in pressure, temperature, as well as cellular damage in the skin. Large and dramatic increases in the activity of nociceptor cells suggests rapid changes in pressure, temperature, or actual cell damage—i.e., a threat to that area. The interneuron cells in the dorsal horn of the spine receive the electrical signals from the nociceptor cells. Their role in the reflex arc consists of processing this information in a quick-and-dirty fashion. Specifically, the interneurons of the dorsal horn sum the incoming electrical signals of nociceptor afferent fibers. If the summed value of the incoming signals from the

nociceptor cells exceeds a threshold level, the interneurons pass activation to efferent motor fibers. One can characterize the information processed by these interneurons as the computation of a threshold function to quickly assess the threat to the skin cells as indicated by the information transmitted by the nociceptor cells, and to determine a differential response based upon that assessment. The final stage of the reflex arc consists of the efferent motor neurons that receive signals from the spinal interneurons and transmit that signal to the muscles, thereby initiating a startle response withdrawal of the area from its current position. Thus, efferent motor neuron activity carries motor response commands to the muscles based upon an inference about the threat to the area.

# 1.5.b Cognitive Science is Interdisciplinary

As the above examples might suggest, theories in cognitive science describe human cognition using information from many different disciplines and using many different categorizations and theoretic models. Not surprisingly, then, cognitive scientists collaborate with one another across disciplines in formulating and integrating theoretic models of various cognitive capacities. For example, in understanding language linguists cooperate with neuroscientists, sociologists, and anthropologists to understand the nature, origins, universal features, and neurological bases on language.

The main disciplines usually cited as having helped to found cognitive science include computer science, neuroscience, philosophy, psychology, linguistics. As the course moves forward, the chapters and text will discuss the basics of each of these disciplines as they relate to cognitive science.

#### 1.5.c Disciplinary Convergence

One central theme that will emerge in the discussions of each of the core disciplines concerns how each of these disciplines converged upon a common conception of cognition and the generalized schema for predicting, retrodicting, manipulating, and explaining cognitive capacities. Each of the disciplinary summaries in the introductory chapters focuses upon those elements of that discipline that contribute to and converge upon this common conception of cognition and cognitive task performance.

## 1.5.d The Central Aim of Cognitive Science

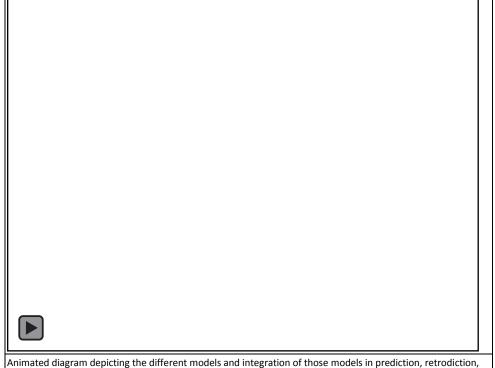
As the material in the course will demonstrate decisively, cognitive scientists have elaborated and expanded the paradigm encapsulated in the above sections to provide tremendous insight into the cognitive task performance of humans and many other animals for an enormous number of cognitive capacities. However, not every aspect of cognition has proven equally and obviously susceptible to such treatments. Cognitive science, of course, seeks to understand all aspects of mentality. As a result, whenever problems arise for the computational paradigm, the text will note these difficulties and current thinking about these areas.

#### 1.5.e Integration of Four Different Theoretical Models

During the course of this chapter and lectures I have portrayed the ways in which cognitive science and cognitive scientists operate in exactly the same manner in which other scientists operate. However, one of the themes that will emerge repeatedly throughout the chapters and associated lectures is that predictions, retrodictions, manipulations, and explanations in cognitive science do differ somewhat from other sciences. Specifically, cognitive scientists predict, manipulate and explain cognitive task performance by creating and illustrating the integration of multiple models for their target system. Moreover, cognitive scientists use these

models and their integration or lack thereof to explain both successful cognitive task performance and unsuccessful task performance.

For now, I will merely outline the four types of theoretic models employed by cognitive scientists and how these models interact in predictions, manipulations, and explanations of cognitive task performance. The diagram below illustrates the four types of models employed by cognitive science and the contributions of each of those models. Firstly, one has the physical theory. In the case of human cognition, the physical theory usually encompasses the neuroscientific account of the relevant neuronal physiology, anatomy, and basic functioning. Thus, in the case of the reflex arc theorists identify the afferent neurons, interneurons, and efferent neurons. Likewise, accounts of basic neuronal functioning indicate how the reflex arc operates. I call this the physical theory. The physical theory does what other physical theories do—it explains how physical work gets done. In looking at the physical theory for the reflex arc, theorists note that the reflex arc treats different levels of afferent neuron input differently. Thus, the system treats different states differently. Cognitive scientists model these changes in the physical system by creating types of states--different categories of states of the components in the physical system--and modeling the system's behavior using these types. I call this the syntactic theory. Thus, theorists also describe the reflex arc in terms of inputs and outputs and the mechanisms by which the reflex arc exhibits a differential response. In employing the syntactic theory, cognitive scientists model the reflex arc in a manner that relates types of activity to other types of activity. As a result, the scientists can now model the physical state changes and the state type changes. Next, theorists associate representational content with the types in the syntactic theory. I call this the representational theory. The representational theory allows scientists to model the state type changes as information transformations.



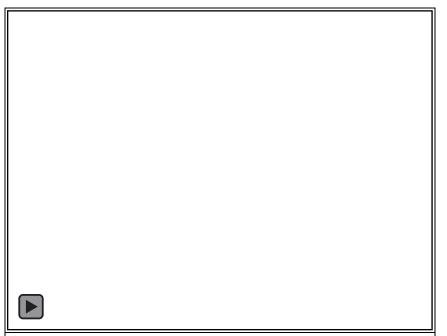
manipulation, and explanation in cognitive science.

In the case of the reflex arc, the actions of the system can now be understood as inferences from nociceptors about changes in pressure, temperature, and/or cell integrity to motor commands issued in response to threats to the affected area. Finally, theorists employ the epistemic theory to model how well and when the system correctly represents the information in the problem domain for the purposes of inference. In the reflex arc, the epistemic theory connects nociceptor functioning to the information relevant to this particular inference. The

epistemic theory, then, shows how the reflex arc tends to function in a manner that correctly responds to real threats to cell integrity in the affected skin areas.

## 1.5.e Understanding When Models Converge and Diverge Cognitive Science as Science

On the view of predictive, manipulative, and explanatory structure advocated in this text and lectures, theoretic models predict and explain both successful cognition and failures of cognition. Theoretic models predict and explain successful cognition by showing how all of the theoretic models accurately predict the target phenomena; The theoretic models have overlapping partial potential models and the target phenomena fail under those partial potential models. Theoretic models explain failures of cognition by showing how one or more of the theoretic models does not predict the phenomena—by showing that the phenomena fall outside the partial potential model for one or more of the theoretic models.



Animated diagram depicting the process of prediction, retrodiction, manipulation, and explanation in cognitive science. Each type of theoretic model has a set of categorizations and relationships that model the phenomena. However, not all the partial potential models for these theoretic models prove coextensive. As a result, when the integration of one or more models fails, one explains failure to perform a cognitive task. Similarly, the integration of all the models allows theorists to explain successful cognitive task performance.

Referring again to the reflex arc, failures of the reflex arc can occur under a number of situations that violate the syntactic, representational, and/or epistemic models, but which the physiological theory still accurately describes. For example, when noxious stimuli rise slowly in intensity the sensory nerve becomes less sensitive through a process called habituation. Likewise, in cases of slowly rising and/or repeated or sustained stimulus the threshold levels in the interneuron rise too, delaying or even erasing the withdrawal response. Communication between the sensory neurons and interneurons can be chemically blocked locally. One can block motor neuron signaling with a paralytic like curare. Many contemporary accounts

of prediction, manipulation, and explanation on suppose all such failures of computational accounts of cognitive functions trace back to the physical theory. However, higher order cognitive causes also lead to divergent partial potential models. For example, conscious inhibitory feedback can prevent the reflex arc from triggering a withdrawal response as well.

#### 1.6 Key Terms

Afferent neurons: Sensory neurons that carry information into the central nervous system (CNS), i.e., towards the spinal cord and/or brain, all fall under the classification of afferent neurons. Discussions of afferent neurons almost always use somatosensory neurons in the skin (proprioceptors, mechanoreceptors providing spatial information about body parts and nociceptors that process pain and temperature). However, rods and cones (the photoreceptor cells in the eye), hair cells in the ear, taste buds, and olfactory receptor neurons.

**Efferent neurons:** All neurons that transmit information from the central nervous system (CNS) to glands and muscles fall under the category of efferent neurons.

**Interneurons:** Interneurons form connections between two or more neurons. So, all the neurons in the CNS are interneurons.

**Paradigm:** Thomas Kuhn uses the concept of a paradigm in two general senses: On the one hand, Kuhn thinks of paradigms as exemplars--specific cases of scientific research having two important properties. Paradigms prove both "sufficiently unprecedented" and highly successful in treating some phenomena. At the same time paradigms exhibit unresolved problems and/or potential, providing the basis for further work to remedy difficulties and extend the central insights. (See *The Structure of Scientific Revolutions* p.10) Kuhn, however, resists any analysis of paradigms into a set of constitutive concepts, theories, instruments, methodologies, values, and similar explicit and tacit commitments. Though Kuhn acknowledges that these elements of paradigmatic cases unite and define practitioners with a given field, he views the tacit nature of these elements together with the variation in understanding and values among individual scientists as rendering such analyses difficult at best and unhelpful at worst.

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