Chapter 4

Arguments

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Chapter Outline

4.1 Introduction: What are Arguments?

The last chapter and lectures explored inferences, characterizing inferences as psychological processes that transform encoded information. We further noted that native human inference abilities are highly contextualized. On other words, human inferential processes rely heavily on the content of the inference and the context in which one makes the inference. Human inference abilities, as psychologists have discovered, fall into two distinct sorts of inference strategies. Psychologists commonly refer to these two inference strategies as System 1 and System 2. The inferences categorized as belonging to System 1 consist largely of innate and highly routinized dispositions that automatically engage when facing a problem. These strategies tend to contextualize a problem, relying upon the specific context and content of the problem. System 1 inference strategies require little conscious awareness and oversight to operate. As a result, these strategies allow for very little conscious access into their functioning and very little conscious oversight of their operations. System 1 strategies tend to share the properties of (a) innateness, (b) automaticity (they work automatically without having to think about or choose them) (c) contextualization (i.e., System 1 inference strategies operate by bringing contextual and content-relevant information to bear on the problem), as well as exhibiting limited conscious (d) awareness, (e) oversight, and (f) insight. On the other hand, System 2 inference strategies involve learned knowledge and techniques. Strategies in System 2 do not automatically engage when a reasoner faces a problem. Indeed, System 2 strategies require conscious awareness and/or oversight to operate, and as such often prove difficult to engage. However, many System 2 strategies tend to compensate for the sorts of weaknesses inherent in System 1 strategies because System 2 strategies tend to embody more decontextualized solution strategies. System 2 inference strategies also provide humans with greater conscious insight and oversight into their inferences. System 2 strategies, as a result, often prove more generally reliable. Finally, we discovered that working memory, the memory subserving conscious awareness and reasoning, severely limits the amount and complexity of information one can process through conscious inference. In this chapter and lectures shifts focus towards arguments. Specifically, this chapter and associated lectures discuss the nature of arguments, their purposes, the different sciences that study arguments, the elements of arguments as studied by logic, the structure of arguments, how to identify and extract arguments from written or verbal language, and how to evaluate arguments using the counterexample method.

4.1.1 Arguments Versus Fights Versus Inferences

This text differentiates arguments from both verbal fights and from inferences. Often times people associate arguments with arguing—that is, yelling, screaming, or other sorts of fractious verbal sparring. For the purposes of this text, yelling, screaming, or other sorts of verbal sparring are not considered arguments. For starters, verbal sparring is largely combative in nature—the goal of verbal sparring consists in winning by defeating one's opponent. Even more refined rhetorical competitions like debates do not count as arguments for the purposes of this class. Many factors may sway the course of a debate—style, mannerisms, the audience's preconceptions, the arguer's willingness to lie, etc.. For instance, most historians portray John Kennedy as the victor in his 1960 debate with Richard Nixon. However, historians do not cite Kennedy's masterful arguments as the reason for his victory. Rather, historians attribute Kennedy's victory to the fact that Kennedy came across better to television audiences than the sweating and awkward Nixon:[1]

> The Kennedy-Nixon debates not only had a major impact on the election's outcome, but ushered in a new era in which crafting a public image and taking advantage of media exposure became essential ingredients of a successful political campaign.

In contrast, arguments serve as mediums for presenting good reasons or accurate evidence in support of a conclusion. So, in the context of critical thinking, arguments are NOT fights.

In similar fashion, this text and associated lectures differentiates arguments from inferences. Inferences transform information inside our heads, generating new information. Arguments present reasons or evidence and draw conclusions much like inferences. However, arguments and inferences are distinct kinds of things. Inferences are psychological processes. Arguments may result from psychological processes and we consume arguments using psychological processes. Nevertheless, arguments themselves are not psychological processes. Arguments do not necessarily relate evidence to conclusions in the same manner that inferential processes transform information. For instance, humans prove quite bad at creating and evaluating many types of arguments.[2-14]

4.1.2 Arguments are Artifacts

What, then, are arguments? Arguments are artifacts – – they are bits of human artifice. Though it may not seem like it; arguments are artifacts in the same way that a computer is an artifact. Like all artifacts, humans create arguments for specific purposes. Moreover, like many artifacts, arguments have sciences devoted to understanding their functioning and improving their manufacture. Just as computers have computer science, arguments have formal logic, probability theory, and statistics. In short, one should think of arguments like one thinks of any other artifact or tool. One might ask, "if arguments are artifacts what purpose do they serve?" Indeed, why do human beings make, exchange, and consume arguments?

4.1.2.a All Artifacts Serve Purposes

People create other artifacts to serve specific purposes—the same holds true for arguments. The primary purpose of an argument is to display evidence or reasons and to relate that evidence or reasons to a specific conclusion. In this way, arguments function as models or presentations of possible inferences. Why model or present inferences to oneself or to others? There are many benefits to modeling or presenting inferences. One benefit common to all uses of arguments lies in their capacity to externalize the structure and content of an inference so that one can free up conscious working memory.

4.1.2.a.1 Arguments as Vehicles for Presentation

The chapter on inferences notes that the advent of spoken language and counting represents a game-changing innovation in how humans to share experiences and insights with one another. Spoken language provides a medium for sharing greater amounts of information and more complex information. Moreover, spoken language potentially facilitates the distillation of the raw information of experiences into a focused presentation of the most important elements. Similarly, the development of counting allows humans to represent events and objects in a manner that makes certain properties explicit, i.e., quantity. Thus, language and counting allow for the sharing of much more complex information over greater distances and longer periods of time. However, the capacity of working memory still places severe limitations on conscious information representation and processing, thereby limiting the amount and complexity of information transmission through spoken languages. When the amount or complexity of information exceeds the capacity of working memory, information is inevitably lost or confused. These limitations of working memory also shape verbal communication.

However, the invention of written languages and number systems dramatically increases human ability to communicate large amounts of complex information, since the consumer of that information now has a stable external representation storing that information. Written language allows people to consume information at a pace that does not overwhelm their conscious working memory. Thus, as the ecosystems chapter notes; written languages and number systems not only externalize the storage of mass amounts of complex information, they thereby allow for much more efficient and extensive manipulation of that information—for instance, through revision, search, and mathematical operations. Arguments provide an excellent example of how human beings use language both verbal and written to communicate large amounts of complex interrelated information like inferences. Likewise, written arguments provide an excellent example of how humans can efficiently manipulate such information. However, while language can mitigate some of the load on working memory, it is not a panacea. As we will see, the best arguments present information in a manner that facilitates understanding. Specifically, both verbal and written arguments organize information in a manner that minimizes the challenges posed by human limitations in processing of large and complex bodies of information.

4.1.2.a.2 Arguments as Vehicles for Persuasion

As noted above, arguments, though not themselves verbal disputes, often appear in verbal disputes because arguments have persuasive power. By presenting evidence in a manner that illustrates how that evidence supports a conclusion an argument can prove persuasive simply by clearly articulating why one should adopt a conclusion. So, well-crafted arguments persuade. But arguments have an additional source of persuasive power. Well-crafted arguments, in the logical sense, have normative power. That is, well-crafted arguments conform to norms for good reasoning and thus inherit the authority of those norms. For example, one type of argument we will study is called a deductive argument. A good deductive argument, called a valid argument, has a structure such that if the premises (evidence) are true the conclusion must also be true. As a result, valid deductive arguments have normative force. If someone presents a valid argument to you with premises that you acknowledge are true, but you deny the conclusion, then you are violating a norm of good reasoning. Indeed, the presenter might legitimately accuse you of acting irrationally. In short, arguments present reasons or evidence and they make claims about the relationship of that evidence to a specific conclusion. Since good arguments conform to norms about reasoning, arguments can prove quite persuasive-- both because of the clarity of the presentation and because the argument has normative force.

4.1.2.a.3 Arguments as Mediums for Preservation

The ability to present a persuasive model of an inference to someone does not, however, exhaust the purposes for which people construct arguments. When one creates an argument-- especially when one writes down that argument-- one creates a more permanent model of an inference one might make in thought. In this way arguments allow people to preserve lines of thought for themselves or for future generations. In other words, one does not have to rely upon one's ability to remember and recall a specific inference once one has committed it to paper or digital media. Indeed, if you go to the library and look in philosophy books or mathematics books you will find lots of arguments. The creators of those arguments preserved those arguments in books for consumption at a later time or in a different place. For example, approximately 2,400 years ago Plato, the ancient Greek philosopher, wrote the following argument in his discussion about religious piety in the *Euthyphro*:[15]

> But if the god-beloved and the pious were the same, my dear Euthyphro, and the pious were loved because it was pious, then the god-beloved would be loved because it was god-beloved, and if the god-beloved was god-beloved because it was loved by the gods, then the pious would also be pious because it was loved by the gods; but now you see that they are in opposite cases as being altogether different from each other: the one is of a nature to be loved because it is loved, the other is loved because it is of a nature to be loved. (p.14)

By writing the *Euthyphro*, Plato encoded his argument in a fashion that preserved it for thousands of years. Moreover, because Plato preserves his thinking about religious piety 2400 years ago students still encounter Plato's argument in philosophy and religious studies classes. Since inferences play such a central role in human cognition, the ability to encode and preserve models of inferences as arguments proves invaluable. Indeed, all of the mathematics that you learned for the last 18+ years consists largely of arguments constructed for the specific purpose of preserving inference patterns for posterity.

4.1.2.a.4 Arguments as Mediums for Increased Comprehension, Analysis, and/or Refinement

The ability to create artifacts that encode models of inferences and thereby free conscious working memory has additional benefits. One benefit of creating external representations of inferential relationships consists in the ability to revise or refine these arguments. As I've noted many times, **arguments depict complex interrelationships-- often times complex interrelationships between large amounts of information.** As result, the amount and complexity of information can make it difficult to formulate an argument. By encoding the argument in an external representation one can free up memory and use that memory to further refine one's argument.

Additionally, and importantly, **when one represents an inference as an argument written on a piece of paper, one frees up working memory. A reasoner can now use working memory to consciously consume, analyze, refine, and/or evaluate that argument.** That is, working memory resources once used simply for storing the argument—for conscious awareness--can now be used to review, refine, analyze, and evaluate the argument. Are connections missing? Are the premises all true? Is the argument deductive or inductive? Does the truth (or high probability) of the evidence make the conclusion true (or more probable)?

As will become clear, even sophisticated, conscientious people can fail to create well-crafted arguments. Nor do all arguments have true or even highly probable evidence. As a result, one important benefit of encoding inferences as arguments consists in the ability to evaluate the quality of the argument in order to determine if it offers adequate reasons for adopting the conclusion. In fact, this chapter helps students learn; (1) how to recognize an argument in a bit of verbal or written language; (2) how to extract an argument from a written or verbal passage; (3) how to craft or revise an argument to make it more effective; and (4) a technique for evaluating one kind of argument—deductive arguments.

4.2 Building Blocks of Arguments

The last section characterizes arguments as artifacts. Specifically, both written and verbal arguments employ language to encode and present evidence or reasons and their relationship to a specific conclusion. In this way, arguments function as models or presentations of possible inferences. Thus, all arguments involve some sort of language into which the arguments are encoded. English, French, Chinese—any language can encode an argument. Indeed, later chapters introduce artificial languages (formal languages) developed to express and evaluate various kinds of arguments. These formal languages provide the means by which the inferential sciences have gained great insights into arguments.

4.2.1 Statements

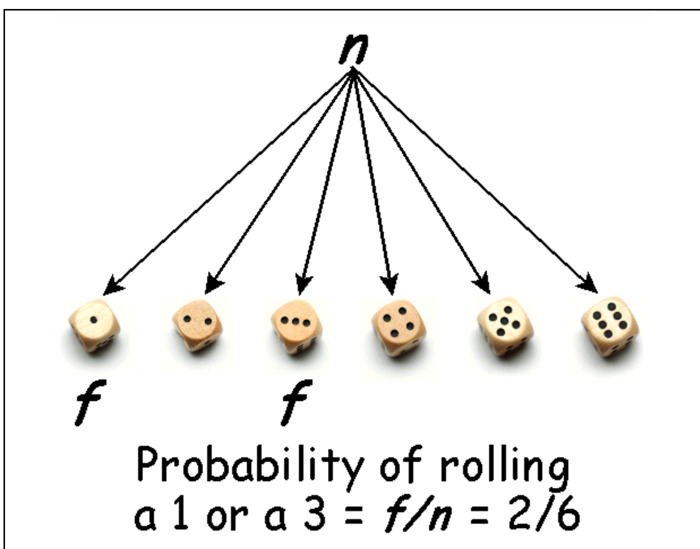
Though all arguments involve language, arguments do not utilize all elements of natural languages. Arguments consist of statements. Statements are linguistic expressions capable of being true or false. However, one need not know the truth or falsity of an expression in order for that expression to count as a statement. One makes a statement within a language by using a declarative sentence or declarative clause within a sentence. One possible exception to this rule is when one formulates a declarative sentence, but that sentence proves semantically meaningless. For instance, Vice President Dan Quayle once said, "The universe is almost infinite." He arguably failed to express a statement since "almost infinite" is an oxymoronic phrase. So, not every expression in a given language qualifies as a statement. Indeed, many expressions in natural language allow humans to communicate without making statements. For example, suppose someone screams, "Look out!" It makes no sense to respond by saying, "I don't think that's true." Warnings, advice, orders, imperatives, proposals, suggestions, and questions all fall under the category of non-statements. To illustrate the difference between statements and non-statements consider the table below:

Statements	**Non-Statements**
Russia has a greater surface area than Pluto.[16-18] (True)	Go look up the surface area of Pluto and Russia. (order/imperative)
Pluto is larger than Eris.[16, 19] (False)	That is amazing! (exclamation/statement of feeling)
It might rain diamonds on Saturn and Jupiter.[20] (True)	Let's go to Saturn to mine diamonds. (proposal/suggestion)
No diamond can conduct electricity.[21] (False)	Don't blue diamonds conduct electricity? (question)
There are more than 30 billion habitable planets in the Milky Way.[22] (Unknown but probable)	Be careful not to overestimate the number of habitable worlds. (warning/advice)
Life on Earth began on another world. (Unknown but improbable)	Life is square and sour. (Meaningless declarative sentence)

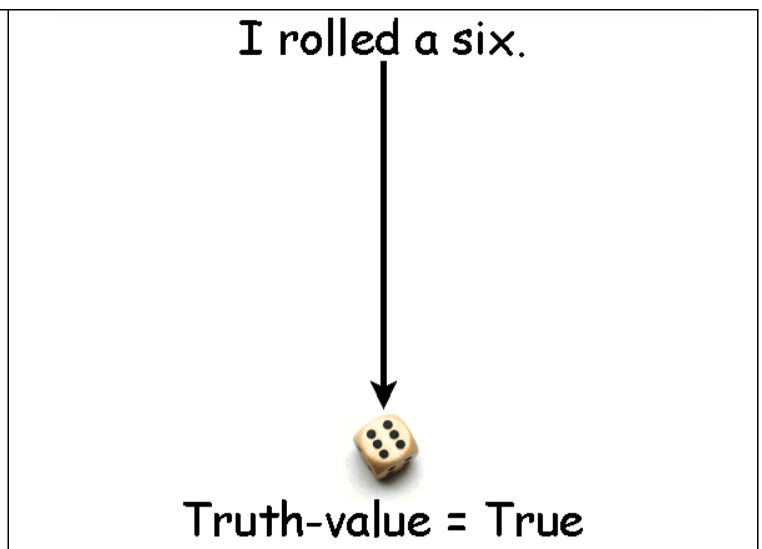
4.2.2 Truth-Values and Probabilities

Statements, as we have seen, are expressions capable of being true or false—of accurately or inaccurately describing the world. One need not know whether a given statement is true or false in order for it to count as a statement, it just needs to describe the world in a factual manner. Because statements express truths or falsehoods about the world, philosophers, logicians, and mathematicians say that statements have truth-values. The truth-value of a statement is just its actual truth or falsity. Consider the expression: "The human body contains trillions of microorganisms — outnumbering human cells 10 to 1."[23] This expression has a truth-value—it is true. Since it has a truth-value it is a statement. Most traditional treatments of deductive arguments suppose that the individual constitutive statements of those arguments have one of two truth values—true or false.

Since statements have truth-values, they also have probabilities. We will revisit probabilities in chapters 9 thru 11. For now, we will understand probabilities as real numerical values ranging from 0 to 1. One can think of probability values as the decimal representation of a fraction, ***f/n***, where ***f*** equals the number of ways the world could turn out such that the statement is true, and ***n*** equals **every** different way the world could turn out. Thus, a probability value represents the likelihood of a given statement being true. A probability of 0 indicates that the truth-value of the statement is false,



Animated movie illustrating the relationship between possible outcomes of rolling a fair die and probability for two different cases. Click on the image to play.



Animated movie illustrating the relationship between actual states of the world and the truth-value of two different statements. Click on the image to play.

i.e., definitely false. Conversely, a probability of 1 indicates that the truth-value of the statement is true, i.e., definitely true. A probability of .25 indicates that out of the four ways the world could turn out, one way makes the statement true (***f/n*** = ¼). On the other hand, a probability of .75 indicates that of the four ways the world could turn out, three ways make the statement true (***f/n*** = ¾). Consider the probability value of the statement; "the value on the fair die after the roll is five." Since there are six different sides on a die, the value for ***n*** = 6. Only one of the sides has five dots, making the value for ***f*** = 1. Therefore, the probability of the statement, "the value on the fair die after the roll is five," equals 1/6 or .167 (***f/n*** = 1/6).

4.3 The Logical Structure of Arguments

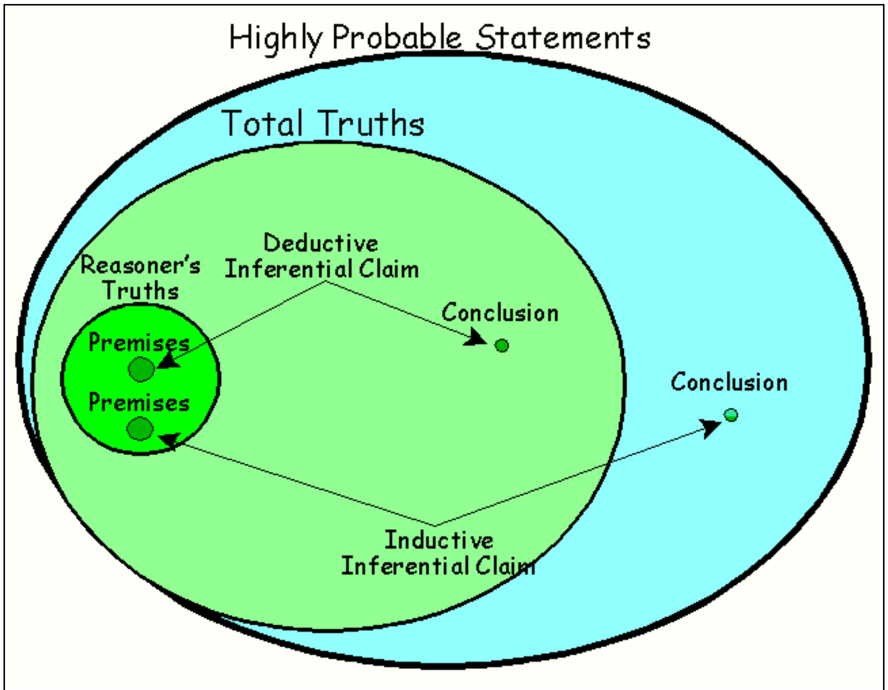
So, arguments are artifacts consisting of collections of statements created for a specific purpose--the modeling of an inferential relationship. Such modeling allows one to present arguments to oneself or others, to persuade others, to preserve a line of thought, as well as to better comprehend, analyze, or refine the modeled inference. Chapter 3 characterizes inferences as psychological processes that take the explicit information available to them and transform that information into new, explicit information. Arguments have a general structure analogous to inferences; arguments relate a set of initial statements to a conclusion statement. To delineate that analogical structure we need to see how individual statements combine within an argument to model a given inference.

4.3.1 Premises

Inferences transform currently explicit and available information to make new information explicit and available. Arguments, on the other hand, display or model a potential information transformation through the assertion of a relationship between two classes of statements--premises and conclusions. So, statements function as explicit and available representations of information in the inference model. Most of the statements in a given argument function as premises, i.e., as the initial information in the inferential process. Thus, premises encode evidence or reasons. Logicians, as a result, often refer to the premises of an argument as the assumptions. The best arguments, therefore, utilize uncontroversial premises (at least uncontroversial in the context in which the argument gets created or used). That is, good arguments start with premises commonly assumed true within the context of the argument. In so far as the creators and/or consumers of an argument question the truth of the premises, the argument proves less effective. So, in creating good arguments one should strive to utilize uncontroversial premises or otherwise support one's premises. Similarly, initial information must be explicit and available to the inferential process. The best arguments will likewise present information in clear, easily comprehensible premises, making that information as explicit and available as possible to the argument's consumers. Conversely, one can challenge an argument by calling one or more of its premises into question. **If one demonstrates the falsity of even a single premise, one undermines the normative force of an argument. One can criticize an argument for having vague or ambiguous premises as well.**

4.3.2 Inferential Claims

An argument seeks to demonstrate or model a relationship between the premises and the conclusion. So, every argument includes an inferential claim—a claim about the relationship between the premises and the conclusion. Most often the inferential claim is implicit in the context of an argument. Nevertheless, every arguer makes an inferential claim. In examining inferences, chapter three distinguishes between two kinds of inferences—deductive and inductive. Arguments likewise fall into deductive and inductive arguments. Thus, one finds that arguers make two general kinds of inferential claims; each claim corresponds to a particular kind of inference. On the one hand, deductive arguments relate statements exclusively in terms of their truth values. Specifically, deductive arguments assert a relationship between the truth of the premises and the truth of the conclusion such that if the premises are true, then the conclusion must also be true. In other words, one can think of the inferential claim in a deductive argument as asserting that if one believes the premises one should also believe the conclusion. Deductive arguments, like deductive inferences, seek to preserve truth. On the other hand, inductive arguments relate the truth-value of the premises to the likelihood of the



Animated movie illustrated the structural relationship between statements in deductive and inductive arguments. Click on image to play animation.

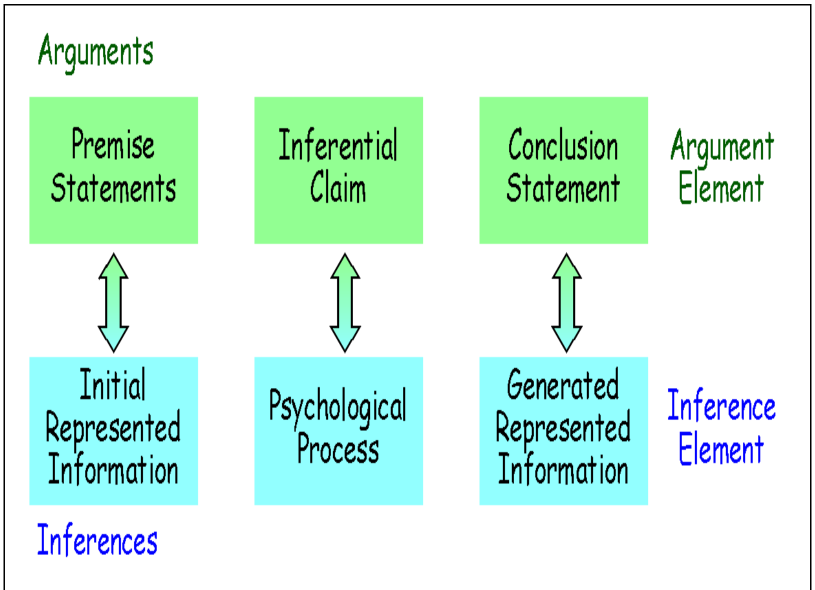
conclusion. Inductive arguments assert that the truth of the premises makes the conclusion highly probable. As a result, inductive arguments involve both truth-values and probabilities whereas deductive arguments involve only truth values. So, inductive arguments assert a relationship between the truth of the premises and the truth of the conclusion such that if the premises are true, then the conclusion must be highly probable. One can think of the inferential claim in an inductive argument as asserting that if one believes the premises, one should believe that the conclusion is very likely to be true as well. Inductive arguments, like inductive inferences, trade a little bit of the guarantee of truth for greater inferential power.

4.3.2 Conclusions

An argument's function consists in establishing the truth or the likelihood of the conclusion on the basis of the premises. Thus, the conclusion expresses the statement that the arguer seeks to establish. However, one needs to understand the inferential claim of the argument in order understand what the argument seeks to establish with regard to the conclusion. As noted above, deductive arguments make a different inferential claim than inductive arguments. Deductive arguments seek to establish the truth of the conclusion. A good deductive argument should make you feel that the conclusion is definitely true. Inductive arguments seek to establish that the conclusion is highly likely to be true—a good bet given the evidence. A good inductive argument should make you feel that you have good reasons to believe that the conclusion is likely to be true. One way to make the contrast between inductive and deductive inferential claims clear is to consider what a good argument tells you about the world—what do you know about the world given the truth of the premises in the argument. In the case of a deductive argument, the truth of the premises guarantees the truth of the conclusion. If the premises of a good deductive argument are true, then the conclusion is also true. The world must be such that the conclusion of that argument is true. In the case of an inductive argument, however, the truth of the premises only establishes that the conclusion is highly probable. That is, the conclusion of a good inductive argument can be highly probable, but false. Consider the following example: The odds of winning the Powerball lottery are 1 in 110,000,000 (f=1 and n=110,000,000 making the probability of a given ticket winning equal to 1/110,000,000). Thus, an argument that takes the odds of winning as its premise and concludes that your particular ticket will not win is a good inductive argument. However, even though the argument does establish that the odds that a given ticket will lose are extremely high, those odds are still consistent with the world turning out such that that ticket wins. Indeed, every winning lottery ticket won despite being highly likely to lose. So, a good inductive argument with true premises can still have a false conclusion.

4.3.4 The Analogous Elements of Arguments and Inferences

I began the discussion of argument structure by stating that arguments model potential inferences. In general a model functions as a representation of entities and their interrelationships. Arguments differ from inferences in that arguments consist of structured collections of static statements. Inferences consist of dynamic psychological processes that transform represented information. Indeed, nothing in an argument corresponds to the intermediate steps of an inferential process. Nevertheless, arguments model inferences because each element of an argument corresponds to an element of an inference in a manner that preserves the important logical relationships. The diagram below illustrates these relationships:



Animated movie depicting the elements of arguments and their corresponding inference elements. Click on image to play animation.

Putting together the elements of arguments presented in the last three sections, one can explicitly show how arguments model inferences by showing how each element in an argument corresponds to an element in an inference. In the diagram (left) the initial explicit and available information for an inference corresponds to the statements comprising an argument's premises. In an argument, the conclusion corresponds to the new explicit and available information generated by the inferential process. During an inference, psychological processes transform the initial information into new explicit and available information. As structured, static collections of statements, arguments cannot model the dynamic changes of an inference process. However, an argument's inferential claim does capture an important aspect of the inferential process: Inferential claims assert a relationship between the truth of the premises and the truth or likely truth of the conclusion. As a result, inferential claims correspond to the truth-functional relationship between the initial explicit and available information in an inference and the generated explicit and available information. Thus, inferential claims capture only the truth-functional relationship between the initial and endpoints of an inference. This truth-functional relationship between initial information and generated information, however, captures much of what makes inferences useful.

4.4 Logic, Probability Theory, and Statistics are the Sciences of Arguments

As noted above, arguments are artifacts crafted for specific purposes. Like other highly specialized artifacts--such as computers—humans have developed sciences to improve human understanding of arguments and to improve techniques for the creation, consumption, and evaluation of arguments. Each category of argument—both deductive and inductive arguments—has given rise to a specialized science. A significant percentage of these lectures and the text familiarize students with some of the basic results and techniques developed as part of the formal treatments of deductive logic, probability theory, and statistics. However, unlike other critical thinking texts instruction in formal techniques of argument extraction, formulation, and evaluation occurs against the backdrop of a thorough explanation of the scientific study of native human inference abilities. The next few sections give students a very schematic history of the development of the sciences of arguments. Specifically, when and where did the sciences of arguments emerge? The exposition begins with the origins of formal logic—the science of deductive arguments--before discussing the beginnings of probability theory and statistics—the science of inductive arguments.

4.4.1 Logic is the Science of Deductive Arguments

The science of deductive arguments is called Formal Logic or Logic. Formal logic has a long history, dating back at least to Ancient Greece and the Greek philosopher Aristotle (384-322 BCE).[24-29] The concept of a proof appears to have developed in ancient Sumerian and Egyptian mathematics, especially geometry, sometime between 3000 and 1650 BCE. The earliest written records of mathematical proofs come from this period. Perhaps the most famous existent written record is called the Rhind Mathematical Papyrus.[30] It dates back to a scribe named Ahmes who copied an older work during the Second Intermediate Period in ancient Egypt.[30-36] By the 11th century BCE the use of standardized proof methods can be found even in medicine.[37, 38] However, historians widely credit Aristotle with both the recognition that the structure of arguments could itself be studied as well as with the invention of categorical logic—a deductive logic we will study later in the term. In a series of works called the *Organon* Aristotle presents treatment of arguments.[24-27] In particular, the *Prior Analytics* Aristotle gives a formal treatment of categorical logic including a basic proof procedure.[39]

Modern logic, particularly logics called the propositional calculus and the predicate calculus, emerged in the late 19th century and early 20th century. Future chapters and lectures discuss propositional logic. Of course, thinking about logic continues after Aristotle. One contribution of note comes from William of Sherwood at some time in the later part of the 13th century.[40, 41] Sherwood wrote *Introductiones in logicam*, one of the influential textbooks during that time.[42] Modern propositional logic has its roots in the thinking of three men: the German mathematician and philosopher Gottfried Wilhelm von Leibniz as well as the British mathematicians George Boole and Augustus De Morgan.[43-46] Scholars identify Leibniz's earliest expression of his mature thoughts in *Generales Inquisitiones de Analysi Notionum et Veritatum,* written in 1686.[47, 48] However, Leibniz's work, while praised by many for its insights, seems to have had little influence on the larger academic world. Instead, historians and logicians point to 139 years after Leibniz, to Boole and De Morgan when discussing the widespread emergence of modern propositional logic. Augustus De Morgan publishes *Formal logic; or, The Calculus of Inference, Necessary and Probable* in 1847.[49] Boole publishes his first logical work that year as well, but follows it with his preferred *An Investigation of the Laws of Thought* in 1854.[50] The American philosopher Charles Sanders Peirce, among others, extends the work of De Morgan and Boole. In 1870 Peirce publishes "Description of a Notation for the Logic of Relatives, Resulting from an Amplification of the Conceptions of Boole's Calculus of Logic."[51] Peirce's paper extends the work of De Morgan and Boole.[51, 52]

Many logic scholars point to a single man and a single work when discussing the invention of predicate logic in much the same manner as scholars point to Aristotle in discussing categorical logic.[28, 53, 54] Gottlob Frege publishes *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens (A Formal Language for Pure Thought Modeled on that of Arithmetic)* in 1879. In *Begriffsschrift* Frege outlines, among many other things, predicate logic. However, Frege himself acknowledges Leibniz in the preface. Moreover, though Frege's work is strikingly original, his project in the work was part of a foundational project within mathematics. Frege certainly benefited from the work of other mathematicians too. Ironically, the *Begriffsschrift* has little influence with other mathematicians and logicians. Eventually, mathematicians and logicians came to understand and appreciate Frege's contributions—though through the unsatisfactory path of the discovery of a flaw in his system by Bertrand Russell.[28, 53-61]

4.4.1.a Logic Studies Deductive Logical Form

Formal Logic, as noted above, is the systematic study of the nature of deductive logical arguments. Since the Aristotle logicians recognize that some arguments have a quality such that if their premises are true, then their conclusions will also always be true. These arguments, as a result, preserve truth. Logicians wondered how these arguments preserve certainty of a true conclusion given true premises. Aristotle is the first known thinker to recognize the mechanism at work in deductive arguments. Aristotle realized that deductive arguments actually have two components. On the one hand, every argument has content—it contains statements that accurately or inaccurately describe the world. But merely formulating an argument with true premises does not guarantee the truth of the conclusion. Consider the following argument:

> The sun is a star.
> The Earth is a planet.
> Therefore, Wallis is the president of the United States.

People acknowledge the uncontroversial truth of each of the premises of the above argument. However, no one finds those premises compelling evidence for the truth of the conclusion. No one—not even Wallis—accepts the truth of the conclusion given the truth of the premises.

Since true premises alone do not seem to guarantee the truth of an argument's conclusion, then some other factor must work in tandem within good deductive arguments such that good deductive arguments can guarantee the truth of the conclusion given the truth of the premises. Aristotle realizes that deductive arguments also have an underlying logical form—a structure that relates content elements to one another such that the truth of the premises guarantees the truth of the conclusion. One way for students to think about logical form is that it acts like the frame of a house: The content of the argument is the façade of the house on this analogy. The façade of a house functions to protect the interior of the house and to make it recognizable and appealing to people. However, beneath the façade a well-built house must also have a well-constructed frame. The frame is difficult to notice when looking at the house. Nevertheless, the frame works to support the necessary relationships between elements of the house so that the façade can serves its function. In this analogy, the logical form of a deductive argument functions within the argument in the same manner as the frame functions in the house. Just as the frame of a house provides the underlying structure that enforces the relationships necessary for the façade to function properly, the logical form of an argument acts within the argument to insure the necessary relationships between the content elements of the argument so that the truth of the premises function to guarantee the truth of the conclusion.

One can also think of the relationship between logical form and an argument's content through a comparison with the relationship between cars and roads. In this comparison the content of an argument functions like a car, as the vehicle for truths. However, a car needs roads and bridges in order to transport its cargo from point A to point B. No matter how wonderful one's car, it still needs a means of connection between its origin and its destination. Likewise, one can have fabulous roads connecting point A to point B, but if one does not have a car, one cannot utilize these connections. An argument's premises might provide one with evidence for a conclusion, but without good logical form to connect the evidence of the premises to the truth of the conclusion, one's premises cannot transport the argument's consumer to the conclusion. Just as different cars and utilize the same roads to get from one place to another, arguments with different content can utilize the same logical form. Likewise, one can formulate an argument with excellent logical form, but if the argument's premises prove false, the argument's consumer cannot follow the truth of the premises to the truth of the conclusion. One can see the relationship between content and logical form by considering the arguments below:

	Good Logical Form	Bad Logical Form
Good Content	Dogs are mammals. Dogs are animals. Thus, dogs are animals and mammals.	Either dogs are mammals or reptiles. Dogs are not reptiles. Thus, dogs are not mammals.
Bad Content	Cats are reptiles. Cats are planets. Thus, cats are reptiles and planets.	Either humans are electrons or cars. Humans are electrons. Thus, humans are cars.

So, every good deductive argument must include two components; good (true) content in its premises **and** a good logical form that guarantees the truth of the conclusion given the truth of the premises. But, Aristotle and his successors made a further, extremely important discovery: While no two arguments can have the exact same content without being about the same thing, many arguments with different content can share the same logical form. Moreover, the shared logical form of arguments functions will always function to guarantee the truth of the conclusion given the truth of the premises--despite differences between argument content. Consider the following four arguments. Each argument has different content and identical logical form:

Either the Earth is a star or the Earth is a planet. (True) The Earth is **not** a star. (True) Therefore, the Earth is a planet. (True.)	Either dogs are mammals or dogs are robots. (True) The dogs are **not** robots. (True) Therefore, dogs are mammals. (True.)
Either Egypt is a country in Africa or Egypt is small mammal. (True) The Egypt is **not** a small mammal. (True) Therefore, Egypt is a country in Africa. (True.)	Either yellow is a color or the Earth is a planet. (True) The yellow is **not** a color. (False) Therefore, the Earth is a planet. (True.)

All of the arguments in the (above) table have good logical form—even the bottom, right argument. However, the bottom, right argument has bad content (a false premise). As a result, the conclusion is not guaranteed to be true given the premises despite the good logical form.

4.4.2 Probability Theory and Statistics are the Sciences of Inductive Arguments (partial)
Inductive arguments fall under the umbrella of probability theory and statistics. The basic framework of the modern mathematical theory of probability dates back to an epistolary collaboration between Gerolamo Cardano, Blaise Pascal, and Pierre de Fermat in 1654.[38, 62-66] Gelermo Cardano's systematic treatment of probability, *Liber de ludo aleae* (*On Casting the Die*), was written in 1654, but not published until after his death in 1663.[67] Christiaan Huygens (1657) followed the three men's work with the first published systematic exposition of probability theory in his *De ratiociniis in ludo aleae* (*On Reasoning in Games of Chance*).[63, 68] Though neither de Fermat nor Pascal ever published works on probability, they are generally thought cited as probability theory's inventors. The works by Cardano and Huygens (particularly the later) provided the systematic foundation upon which future mathematicians continue to work.

One can find isolated uses of statistics in history, for instance in early cryptography. The first known published manuscript in the area dates back to Al-Kindi an Muslim mathematician from Baghdad who published a book called, *Manuscript on Deciphering Cryptographic Messages*.[69, 70] Modern treatments of mathematical statistics arguably begins with the work of John Graunt and William Petty in 1662.[71, 72] Systematic treatments of statistics as well of many of the key concepts do not begin to emerge until around 1810.[72-74]

4.4.2.a Probability and Statistics Study Inductive Logical Form

4.5 Identifying, Extracting, Optimizing, and Evaluating Arguments
So far this chapter and associated lectures present factual information about arguments. Utilizing that information to effectively traffic in arguments requires certain skills as well as factual knowledge. Thus, the next few sections turn to three important skills: (1) the skill of identifying an argument in a bit of written or verbal language, (2) the skill of extracting an argument from a bit of verbal or written language, (3) rendering that argument in its optimal form, called "standard form." As we will see, the ability to identify, extract, and optimize arguments turns out to be a crucially important skill-set for any reasoner.

4.5.1 Uses and Benefits
The skill of identifying and extracting arguments has many benefits. To start, every arguer seeks to present the argument's consumers with good reasons to accept the conclusion as true or highly probable. The ability to recognize when a person seeks to provide you reasons to accept some statement can help you to better understand their intent. Likewise, arguments give a person insight into the arguer's reasoning and world view. As a result, understanding a person's arguments helps one to better understand the person.

Of course, people do not always create the argument that would best serve their purpose. Identifying and extracting arguments allows one to make their argument explicit and to render their argument in its most compelling form. To this end we will practice creating versions of an extracted argument that are in standard form. Recall that standard form arguments are arguments that present the premises and conclusions in a language and order that makes the argument as clear and as forceful as possible. One might think that the arguer has sole responsibility for making their argument as good as possible. One might even think, "Why help someone argue for something I don't believe?!" However, I would suggest that such thoughts stem from a misconception of the nature of arguments. Recall that I began the chapter by distinguishing arguments from verbal fights or debates. The goal of argumentation should consist in communicating information and insight. Indeed, it does one little good to win an argument but to continue to believe a falsehood. One should always engage in argumentation with the goal of sharing information, understanding alternative perspectives, and benefiting from another's insight. One might not accept the conclusion of someone's argument, but one still learns and benefits from understanding the arguer's reasons and why those reasons proved inadequate. Conversely, one may well discover that the arguer is right, or that they have a perspective on the conclusion that you had not considered. You might change your mind about the conclusion or gain insight into the complexities of the issue. Identifying and extracting an argument allows for the sort of careful analysis that facilitates such benefits in that it results in one's creating a more rigorous, easily comprehendible, and impactful statement of yours or another's reasoning.

Indeed, just as writing or revising a term paper often gives one greater insight into the material than simply reading about the topic, identifying and extracting yours or another's arguments allows you to better comprehend connections between ideas and issues. Because arguments explicitly seek to draw connections between premises and conclusions—between evidence and other statements—identifying and extracting arguments almost always results in a deeper and more thorough understanding of a given topic.

Finally, identifying and extracting an argument from a bit of language allows one give the argument an explicit representation. Such representations facilitate consumption of the argument and evaluation of the argument. This chapter ends with a technique for evaluating the goodness of deductive arguments called the counterexample method. The counterexample method allows one to evaluate a deductive inferential claim. Deductive arguments that do not establish the claimed relationship between the truth of the premises and the truth of the conclusion are flawed arguments. The counterexample method allows one to evaluate whether the deductive inferential claim really does hold between the premises and the conclusion. If the inferential claim fails to hold, then the argument cannot establish its conclusion—even with true premises!

4.5.2 Challenges

So, there are numerous benefits to identifying and extracting arguments. But, one might think, since arguments are artifacts created specifically for the purpose of relating premises to conclusions in accordance with specific inferential claims, identifying and extracting arguments must prove relatively unproblematic. Unfortunately, argument identification and extraction proves more difficult than it might initially appear. Arguments are just bits of language, and just like other bits of language--some bits of argument turn out better suited to their purpose than others. Everyone has misspoken or uttered something that was ungrammatical. As it is for individual statements, so it is for those collections of statements we call arguments. Indeed, many arguments get created on the fly. Even when one deliberately and carefully creates and argument, one sometimes fails to render an argument in its most effective form. Among the common problems one finds in arguments are the following: Material is missing from the argument. Sometimes one omits or forgets an important bit of information. One might simply assume that people have information in their mind and can make the connections in the argument. Such missing information can prove problematic when people bring different world views to an argumentative context. Likewise, the argumentative context might not prove conducive to the highlighting certain information for the argument's consumer. Sometimes from within their own context the arguer sees the conclusion as so clearly established by the argument that he or she omits the conclusion from the argument.

4.5.3 Standard Form Arguments

So far we have seen that arguments are artifacts constructed from individual statements. An arguer creates these individual statements and arranges them so that they have a particular structure within an argument. Some of the statements serve as premises and one of the statements serves as a conclusion. Additionally, the argument implicitly or explicitly asserts a relationship between the premises and the conclusions called the inferential claim. Deductive inferential claims assert that if the premises are true, the conclusion must also be true. Inductive inferential claims assert that if the premises are true, then the conclusion must be highly probable. However, consider the following arguments:

> It is right that men should value the soul rather than the body; for perfection of the soul corrects the inferiority of the body, but physical strength without intelligence does nothing to improve the mind. --Democritus[75, 76]

And

> Unlimited tolerance must lead to the disappearance of tolerance. If we extend unlimited tolerance even to those who are intolerant, if we are not prepared to defend a tolerant society against the onslaught of the intolerant, then the tolerant will be destroyed, and tolerance with them. –Karl Popper,[77, 78] *The Open Society and Its Enemies*[79]

And

> For any question, either you know the answer or you don't. If you know the answer, then inquiry is unnecessary. If you don't know the answer, you'll have no way of recognizing the correct answer when it presents itself — for if you don't know what the correct answer is, how will you distinguish it from false answers? So if you don't know the answer, inquiry is impossible. Plato[80, 81], *The Phaedo*[82]

And

> There cannot be any emptiness; for what is empty is nothing, and what is nothing cannot be. – Melissus of Samos[83, 84]

Each of the above arguments has premises and a conclusion. Each has an implicit inferential claim. However, their overall structure differs dramatically. Some of the arguments begin with the conclusion. Other arguments have the conclusion at the end. Some premises get expressed in separate sentences, while others get combined into more complex sentences.

While the authors of these arguments commit no errors, they could still improve their arguments. Remember from the inferences chapter that humans face two challenges when making inferences and decisions. On the one hand, content and context can unduly influence one's inferences. On the other hand, humans face severe limitations on the amount of information that they can process in working memory. As a result, when modeling an inference in an argument one ought to work hard to help the argument's consumer overcome these obstacles to comprehending and appreciating the argument. Since content can sometimes obscure or override the underlying logical form of an argument, one ought to craft one's premises and conclusions so that the important underlying logical structure is as salient as possible. To further facilitate comprehension, one ought to express each premise in a clear, concise sentence. The best arguments present their premises and conclusions in concise, but easy to understand language. Since the truth of the premises helps to establish the truth of the conclusion, one should draw one's premises, whenever possible, from commonly held or uncontroversial information. When premises express more controversial opinions, one should provide additional support for those premises.

Logicians refer to arguments that adhere to conventions that facilitate easy comprehension as "standard form arguments," or arguments in standard form. Standard form arguments follow four rules:

1.) Terms used in the argument are always used with the same meaning.
2.) Each premise is expressed in a single declarative statement.
3.) The arguer presents the premises first and the conclusion last.
4.) The premises are ordered so that relationships between ideas flow from one premise to the next.

Given these rules, let's revisit the arguments above starting with the argument from the ancient Greek philosopher Melissus of Samos:

Melissus Argument Original Version:
There cannot be any emptiness; for what is empty is nothing, and what is nothing cannot be. –Melissus

Melissus Argument Standard Form Version:
What is empty contains nothing.
Emptiness is nothing.
What is nothing cannot exist.
Therefore, emptiness cannot exist.

Note that in revising Melissus' argument I make several changes. To start, I write the conclusion as a separate declarative sentence and move the revised conclusion from the beginning to the end of the argument. Each of the premises are also written as separate declarative sentences and placed at the beginning of the argument. To clarify the identification of emptiness and nothing, I include an intermediate premise. Finally, I express the premises in clear, concise language in which the same ideas get expressed using the same words and phrase structure.

Democritus Argument Original Version:
It is right that men should value the soul rather than the body; for perfection of the soul corrects the inferiority of the body, but physical strength without intelligence does nothing to improve the mind. –Democritus

Democritus Argument Standard form version:
Strength of mind, i.e., intelligence, also improves inferiority of the body.
Physical strength without intelligence does nothing to improve the mind.
Improving the mind has both physical and mental benefits while improving the body does not.
As a result, people should value and cultivate the mind over the body.

In revising Democritus' argument I again make several changes. I write the conclusion as a separate declarative sentence and move the revised conclusion from the beginning to the end of the argument. I again write each of the premises as separate declarative sentences and place them at the beginning of the argument. To clarify the identification of the soul with the intellect in Greek culture, I replace mention of the soul with references to the mind. Overall, I strive to express the premises in clear, concise language in which the same ideas get expressed using the same words and phrase structure. I include original and standard form versions of Plato's and Popper's arguments without further comment below.

Plato Argument Original version:

For any question, either you know the answer or you don't. If you know the answer, then inquiry is unnecessary. If you don't know the answer, you'll have no way of recognizing the correct answer when it presents itself — for if you don't know what the correct answer is, how will you distinguish it from false answers? So if you don't know the answer, inquiry is impossible. Plato, *The Phaedo*

Plato Argument Standard form version:

For any question, either you know the answer or you don't.
If you know the correct answer, then inquiry is unnecessary.
If you don't know the correct answer, then you cannot distinguish correct from incorrect answers.
If you cannot distinguish correct from incorrect answers, then you'll have no way of recognizing the correct answer when it presents itself.
If you have no way of recognizing the correct answer when it presents itself, then inquiry is impossible.
Therefore, if you don't know the correct answer, successful inquiry is impossible.

Popper Argument Original Version:

Unlimited tolerance must lead to the disappearance of tolerance. If we extend unlimited tolerance even to those who are intolerant, if we are not prepared to defend a tolerant society against the onslaught of the intolerant, then the tolerant will be destroyed, and tolerance with them. –Karl Popper, *The Open Society and Its Enemies*

Popper Argument Standard Form Version:

If we extend unlimited tolerance even to those who are intolerant, then we will not be prepared to defend a tolerant society against the onslaught of the intolerant.
If we are not prepared to defend a tolerant society against the onslaught of the intolerant, then the tolerant will be destroyed and tolerance with them.
Thus, unlimited tolerance must lead to the disappearance of tolerance.

4.6 A Four Step Method for Identifying and Extracting Arguments

4.6.1 Identifying an Argument

4.6.1.a The Goal of Argument Identification

4.6.1.b Context

4.6.1.c Logical Form Words

When crafting arguments an author will often use specific words or phrases to indicate the role of a specific statement in the overall structure of an argument. These words and phrases fall into two categories: premise indicators and conclusion indicators. As one might suspect, premise indicators mark the presence of a premise; conclusion indicators mark the presence of a conclusion. Thus, one can use the presence of premise and/or conclusion indicators as a heuristic device for identifying potential arguments in both spoken and written passages. The table below provides a list of words and phrases commonly used as premise indicators. Students should try to familiarize themselves with all these words. Though the chapter provides extensive lists of premise and conclusion indicators, students should not feel overwhelmed or intimidated by these lists. For one thing, students can also rely to some extent upon their competence in English to recognize words and phrases that function as premise and conclusion indicators in a passage. Moreover, as students practice and gain facility in the skill of identifying arguments, their linguistic intuitions will also be refined.

Some Common Words and Phrases in English that Indicate the Presence of Statements Functioning as Premises		
as	follows from	insofar
as indicated by	for	in that
as shown by	for one thing	may be inferred from
assuming	for the reason that	on the assumption that
assuming that	given	owing to
because	given that	seeing that
considering that	inasmuch as	since
due to	in light of	whereas

The sentences that follow illustrate some of the above words and phrases functioning as premise indicators. The premise indicators are in bold and the premises they indicate are in green.

You will have difficulty doing well on the test, **as** you haven't studied.
A flower is a reproductive organ, **as shown by** the presence of pollen at the anther and eggs at the stigma.
Assuming that the weather models are accurate, we can expect greater numbers of record high temperatures.
Insofar as the data are correct, one can conclude that the data support the hypothesis.
Considering that scientists have documented evidence in the fossil record from all over the planet, evolution has tremendous empirical support.
This figure is a square, **for** it is a three-sided, enclosed planar figure.
This figure is a square **inasmuch as** in is a four-sided enclosed planar figure.
Seeing that you are younger than twenty-five, you have a low risk of a heart attack.
On the assumption that you are in Canada this Looney has a value of one dollar.
A diagnosis of HIV positive status **may be inferred from** a positive blood test.
Whereas some Americans support the war in Afghanistan, the vast majority do not.

One can see how the words and phrases listed above often function to indicate that a statement expresses a premise in an argument. Other words and phrases function to indicate that a statement expresses the conclusion of an argument. The table below provides a list of words and phrases commonly used as conclusion indicators in English.

Some Common Words and Phrases in English that Indicate the Presence of Statements Functioning as Conclusions		
accordingly	Implies; implies that	so
as a result	in conclusion	whence
clearly	in consequent	wherefore
consequently	it follows; it follows that	which entails that
demonstrates that	it must be that	which proves that
entails that	must be the case that	which implies that
ergo	necessarily	which means that
follows that	therefore	we can conclude that
for this reason	thus	we may conclude
hence	shows that	we may infer

The sentences that follow illustrate some of the above words and phrases functioning as conclusion indicators. The conclusion indicator is in bold and the conclusion it indicates is blue.

Global warming **implies that** ocean water levels will rise.
I think, **therefore** I am.
Necessarily, two plus two equals four.

It is November 4, **ergo** it is Election Day.
I am human; **consequently**, I am mortal.
Fido is a dog, **so** Fido is a mammal.
Julie is 21, **which means that** Julie can legally drink alcohol in California.
We may infer that this sentence illustrates a conclusion indicator.
This animal is a fish; **hence**, this animal can swim.

Though premise and conclusion indicators can prove very useful as superficial grammatical signs of argumentation in a bit of spoken language or a written passage, indicator words do not always function in this capacity. These same phrases often function as what English professors call transitional words and phrases. Transitional phrases serve to connect the elements of an exposition so as to ease the flow of the exposition and in order to impose a narrative structure. For example, the word "since" can indicate a statement expresses a premise in an argument. However, "since" can also function to indicate a temporal relationship or to impose a temporal structure upon an exposition. The premise indicator, "as," also has a transitional phrase function of indicating a comparison. Consider the following statements:

Since: Premise Indication
You have a dollar, since you have four quarters.

Since: Temporal Structure
Humans have had a presence in space since the 1950s.

As: Premise Indication
You will have difficulty doing well on the test, **as** you haven't studied.

As: Comparison Structure
Many aircraft carriers are as long **as** the empire state building is tall.

So, premise and conclusion indicators may appear in a passage, yet function as transitional words or phrases that do not indicate the presence of an argument. Moreover, arguers can easily express premises and conclusions without indicator words, meaning passages without any indicator words may well contain arguments. As a result, looking for premise and conclusion indicators can only provide students with a heuristic strategy for identifying arguments in spoken or written passages. Put another way, when one notices one or more premise or conclusion indicators in a passage, one should carefully consider whether the passage contains an argument. The presence of such words and phrases does not insure the presence of an argument. Likewise, the absence of premise and conclusion indicators does not preclude the presence of an argument in a passage.

4.6.2 Understanding and Formalizing the Argument
4.6.2.a Identify the Premises and Conclusion
4.6.2.b Identify the Inferential Claim
4.6.2.c Rephrase the Argument to Make the Structure Apparent
4.6.2.d Eliminate Extraneous Material
4.6.2.e Provide Missing Elements
4.6.3 Evaluating a Deductive Argument's Logical Form
4.6.3.a The Counterexample Method
4.6.3.a.1 Rendering the Argument's Logical Form
4.6.3.a.1 Create a Substitution Instance
4.6.4 Evaluating the Argument's Content

4.7 Summary

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